A model for the DC characteristics of a laser diode

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Abstract—Looking for a self-consistent formula for the DC P(I) characteristics of a laser diode (LD), able also to predict the threshold current, a model has been developed for the general electric and optical DC characteristics of a forward-biased LD.

I. INTRODUCTION

The paper was originated by the attempt to answer a “simple” question, proposed to the Author by some colleagues involved in the development of external-cavity laser diodes. They presented the experimental P_OUT(I) curves of one of such devices, obtained by changing the total optical losses, and asked the way to predict the associated changes of both the threshold current and the optical efficiency. The curves duly followed one of the most popular formulas (eq.(1)) in laser diode engineering, that relates the output power P_OUT to the injected current I after defining among the input parameters the threshold current Ith, the quantum efficiency ηq, the internal losses αi and the mirror losses αm:

\[ P_{OUT}(\nu) = \begin{cases} 
\frac{I - I_{th}}{q} \eta_{q} h \nu \alpha_{m} / (\alpha_{i} + \alpha_{m}), & I > I_{th} \\
0, & I \leq I_{th}
\end{cases} \quad [1]
\]

The other two quantities in eq.(1) are the electron charge q and the peak photon energy hν.

It is an excellent formula for practical uses, and is reported in various forms in any textbook on solid state lasers [1]. It summarizes the most peculiar physical and technological properties of such a device, at least in the steady state: the highly monochromatic emission, the existence of a dramatic transition that leads the diode from negligible to strong emission, the electrical pumping, the presence of a resonant cavity, the competition of the light generation process with optical losses and with other non-radiative processes.

Unfortunately, it asks for an external definition of Ith, which makes it useless for answering our initial question, and also in many device and system design steps and even in most of the reliability qualification tests.

For instance, it is impossible to use eq.1 to predict the influence of a different reflection coefficient for one or both of the mirror facets (variation of αm), or of the onset of defects inside the active region (reduction of ηq), or of another composition of the active layer (different hν), or even of a different thickness of the same layer (influence on the confinement factor, and then on the internal loss term αi), or, finally, of the different engineering of the external cavity in devices of such technology (variation of the loss term, again). The threshold current Ith is affected in each and all the above cases, and it even may change itself without affecting any of the above listed quantities (as for anomalous charge recombination, i.e. at growing defects, outside the active layer). In any case, eq.1 does not work.

The paper aims to replace eq.1 with a single expression, able to include the threshold condition inside its predictions. It will be shown that the new equation embeds and confirms eq.1, as worldwide experience would require. The diode voltage will play an important role, as a new parameter to be monitored, not only to drive a laser device, but also to have a different insight on concepts, as optical gain (and gain saturation), transparency threshold, detailed balance of the spontaneous and stimulated emission, and absorption rates, that eq.1 hides for the sake of simplicity. It will be a treatment deeply-rooted on the standard laser theory, but addressed mostly to electrical engineers, to give a more familiar device-oriented point of view on at least the fundamental DC characteristics of a laser diode.

II. THE LASER MODEL

A. Radiating, recombination and Shockley components

The laser diode is modelled as a pipe with uniform section S flood by electrons and holes from the n and p sides, respectively. A Double Heterostructure (DH) is assumed that implies a thin epitaxial active region set inside the much wider depletion layer of the diode.

The total current I flowing across the device is considered made of two main components: the recombination current Ia due to all and sole the recombination events (of any kind) that
translates into the total emitted optical power upon leaving the active region per unit time. This, in turn, destroyed, this number is also the number of photons region to produce light per unit time. Because all of them are on the left side, the number of charges that enter the active region; at which the separation of the quasi-Fermi levels $E_{Fa}$ and $E_{Fp}$, that is of the charge injection level:

$$qV = E_{Fa} - E_{Fp}$$

Here, any ohmic effect has been neglected or, better, has been left to the separate definition of the overall series resistance $R_n$ that links the voltage $V$ in eq.4 to the external applied and measured voltage $V_a$ and to the total current $I$ by:

$$V_a = V + R_a I$$

The recombination current $I_r$ includes all the radiative events inside the active region. Let us suppose that they give account for a fraction $\eta_r$ of the total number of recombination events in that region (which makes $\eta_r$ exactly the quantum efficiency that appears in eq.1). We can accordingly define a current $I_{sh}$ that is completely consumed in creating photons inside the active region:

$$I_{sh} = \eta_r I_a = \eta_r (1 - I_{sh})$$

If now we divide eq.(5) by the electron charge $q$, we obtain, on the left side, the number of charges that enter the active region to produce light per unit time. Because all of them are there destroyed, this number is also the number of photons leaving the active region per unit time. This, in turn, transforms into the total emitted optical power upon multiplication by the peak (or mean) photon energy $h\nu$. Finally, if a fraction is considered of the total optical power that is proportional to the ratio between the only mirror losses $\alpha_m$ and the total losses $\alpha_T = \alpha_r + \alpha_m$, then we have exactly the useful optical power that we call $P_{OUT}$:

$$P_{OUT} = \frac{I_{sh} h \nu \alpha_m}{\alpha_r + \alpha_m} = \frac{1 - I_{sh}}{q} \eta_r h \nu \frac{\alpha_m}{\alpha_r + \alpha_m}$$

Eq.(6) is just a little bit more general than eq.(1): it relates $P_{OUT}$ to both the current $I$ and the voltage $V$ (through eq.(3)), that of course are not independent. We need one more relation, able to explicitly express $P_{OUT}$ as a function of $V$ in order or to eliminate $V$ and obtain $P_{OUT}(I)$ in closed form or, maybe even better, to use the voltage $V$ itself as both a physical quantity and a mathematical parameter to simultaneously describe $P_{OUT}(V)$ and $I(V)$. In that case, eq.(6) will be really a good result.

B. The Photon Density and $P_{OUT}$

The rate equation for photons at frequency $\nu$ inside the active region relates the time variation of the spectral photon density $\phi_\nu$ to the rates of spontaneous emission $R_{sp}$, stimulated emission $R_s$ and absorption $R_{abs}$ and to the total rate of optical loss that is proportional to the photon density $\phi_\nu$, itself through a characteristic time that often is indicated as the cavity time $\tau_c$:

$$\frac{\partial \phi_\nu}{\partial t} = R_{sp}(\nu) + R_s(\nu) - R_{abs}(\nu) - \frac{\phi_\nu}{\tau_c}$$

The cavity time is another way to describe the total optical losses $\alpha_T$ For practical purposes, the equivalence:

$$\alpha_T = 1/c \tau_c$$

is often useful.

The photon generation and absorption events correspond to photon-assisted electron-hole recombination and generation, respectively. For photons at a given energy $h\nu$, they are related to the densities of electrons $n_e$ and holes $p_h$ that share the same momentum $k$ and whose respective energies $E_n$ and $E_p$ are just separated by $h\nu$:

$$E_n - E_p = h \nu , k(E_n) = k(E_p)$$

and, for the sole photon-stimulated transitions, also to the photon density $\phi_\nu$:

$$\begin{align*}
R_s(\nu) & = A \phi_\nu n_e \\
R_{abs}(\nu) & = B \phi_\nu p_h
\end{align*}$$

The proportionality constants $A$ and $B$ assume the role of the Einstein coefficients, while the dashed densities $\bar{n}_e$ and $\bar{p}_h$ indicate the complementary densities, that is the density of filled states in the valence band at energy $E_p$ for $\bar{n}_e$, and the density of empty states at energy $E_n$ for $\bar{p}_h$, when eq.(9) holds.

It is easy to see that, because of eq.(4) and (9), the Fermi-Dirac expressions for the charge densities lead to:

$$\begin{align*}
p_e n_e & = g_e \left[ \exp \left( \frac{h \nu - qV}{2kT} \right) + 1 \right]^{-1} \\
p_h p_h & = g_h \left[ \exp \left( \frac{h \nu - qV}{kT} \right) \right]
\end{align*}$$

where the symbol $g_e^2$ indicates the product of the density of states of electrons at energy $E_n$ and of holes at energy $E_p$ when eq.(9) holds:

$$g_e^2 = g_e(E_n) g_h(E_p)$$

The appearance of the applied voltage $V$ is a direct consequence of eq.(4).

For the steady state $\frac{\partial \phi_\nu}{\partial t} = 0$, eq.(7) may be solved for $\phi_\nu$:

$$\phi_\nu = \frac{A}{B} \left[ \exp \left( \frac{h \nu - qV}{2kT} \right) + 1 \right] + \frac{B \tau_c g_e^2}{\exp \left( \frac{h \nu - qV}{kT} \right) - 1}$$
In the ideal case of no losses ($r_c = \infty$) at equilibrium ($V = 0$) and no forbidden frequencies ($g_2^2 \neq 0$ for any frequency $v$), the photon density must coincide with that of a black body:

$$\phi v^2 \bigg|_{n=0, \infty} = \frac{8\pi \nu^2}{c^3} \exp \left( \frac{\nu}{kT} \right)^{-1}$$

from which we obtain: $A = \frac{\nu^2}{B} \frac{8\pi c^2}{B} c^3$, and then:

$$\phi v^2 = \frac{8\pi \nu^2}{c^3} \alpha_\nu \exp \left( \frac{\nu - qV}{2kT} \right)^{-1} \Omega$$

where we put:

$$B \tau_c g_2^2 = \Omega / \alpha_\nu, \quad \Omega = B g_2^2 / c \quad \alpha_\nu = 1 / \tau_c$$

and we should recall that both $\Omega$ and $\alpha_\nu$ are functions of the frequency $v$ because of the joint density of states $g_2^2$, but also because of the cavity time $\tau_c$ that, in turn, depends on cavity resonance.

The link between $\phi v^2$ and $P_{OUT}$ derives from the integration over the frequency $v$ of eq.(7) for the steady state:

$$\int \frac{\phi v^2}{\tau_c} dv = \left[ R_{\nu} (v) + R_{\nu} (v) - R_{\nu v0} (v) \right] dv$$

On the right side, we have the net integral photon generation rate, that is also the net total rate of radiative electron-hole recombination, that is in turn simply connected to $I_{ph}$ (eq.5) by dividing the latter by the electron charge $q$ and the volume $S d$ of the active layer whose area is $S$ and whose thickness is $d$.

$$\frac{I_{ph}}{q} = \frac{S d}{\tau_c} \int \phi v^2 dv$$

Recalling now eq.(5) and eq.(6):

$$P_{OUT} = \frac{S d}{\tau_c} h v \frac{\alpha_\nu}{\alpha_\nu + \alpha_m} \int \phi v^2 dv$$

Eq.(15) formally links, through the explicit expression in eq.(12), $P_{OUT}$ to the voltage $V$, and its practical use depends on the possibility to calculate the integral.

III. THE $P_{OUT}(I)$ CHARACTERISTICS FOR THE SINGLE-MODE

Equation 1 is obviously written for the single-mode operation, that may be described in the present model by accepting a single fixed value for the photon energy $h v_0$, and consequently, from eq.(14):

$$I_{ph} = \frac{S d}{\tau_c} \int \phi v^2 dv = \frac{S d}{\tau_c} \int \phi v^2 \left( \frac{v}{v_0} - 1 \right) dv$$

$$= \frac{I_0 \alpha_\nu}{\Omega v_0} \left[ \exp \left( \frac{h v_0 - qV}{2kT} \right)^{-1} + \Omega v_0 \exp \left( \frac{h v_0 - qV}{2kT} \right)^{-1} \right]$$

where we put:

$$\frac{S d q \pi v_0^3}{\tau_c} = I_0 \alpha_\nu, \quad I_0 = S d q \pi v_0^3 / c^2$$

that is a constant with the dimensions of a current.

Eq.(16) defines the electrical characteristics $I_{ph}(V)$ of the pure “radiative device” that for low values of $V$ behaves as a normal Shockley diode:

$$I_{ph} = \int \frac{\pi v_0^3}{\tau_c} \phi v^2 dv = \int \frac{\pi v_0^3}{\tau_c} \phi v^2 \left( v / v_0 - 1 \right) dv$$

but runs towards a dramatic transition as $qV$ increases (fig.1). There is, indeed a limiting value $qV_{th}$ that leads the denominator in eq.(16) to vanish:

$$qV_{th} = h v_0 / kT \ln \left( \frac{\Omega v_0 + \alpha_\nu}{\Omega v_0 - \alpha_\nu} \right)^2$$

Approaching that threshold value of the applied voltage, the radiative current $I_{ph}$, and then the overall current $I$, rises faster and faster, and any further current increase corresponds to more and more negligible variations of $V$: it is the voltage clamping that is experimentally observed in any laser diode (after subtracting the ohmic contributions) and that witnesses the “freezing” of the quasi-Fermi levels at the firing of the laser effect.

The practical experience that gives $qV_{th} \approx h v_0$, within some fraction of $kT$, indicates that current values for $\Omega v_0$ are large in comparison with $\alpha_\nu$. 
Now, $I_{ph}$ has been defined, in eq.(2) and (5), as only one of the components of the total current $I$. Its proportionality to $I_d$, as stated by eq.(5) surely holds when the laser emission takes place, because it is the base of the definition of the quantum efficiency, as it appears since eq.(1).

What is important to note is that the other component, the Shockley current that appears since eq.(2) and (3), is by far not negligible at low injection levels (below threshold), simply because otherwise, if $I_{sh}<<I_{ph}$ for any injection level, then the $P_{OUT}(I)$ would be a straight line, which is not. On the contrary, $I_{sh}>>I_{ph}$ until the voltage achieves the clamping value (fig.3)

It follows that the threshold current may be safely estimated as that value that $I_{sh}(V)$ assumes for $V=V_{th}$, as defined in eq.(19):

$$I_{th} = I_{sh}(V_{th}) = I_{sh} \exp \left(\frac{qV_{th}}{kT}\right) = I_{sh} \exp \left(\frac{hV_0}{kT} \left(\frac{\Omega_v + \alpha_T}{\Omega_v - \alpha_T}\right)^2\right)$$

Now, we can use eq.(16) to express $qV$ as a function of $I_{ph}$, and substitute into eq. (3) to obtain $I_{sh}$ as a function first of $I_{ph}$ and then, from eq.(6), of $P_{OUT}$. Eq.(6) now reads:

$$P_{OUT} = \eta \frac{hV_0}{q} \alpha_T \left[ I - I_{sh} \left(\frac{\Omega_v + \alpha_T}{\Omega_v \left[1 + \frac{\Omega_v + \alpha_T}{\Omega_v} \left(\frac{I_{ph}hV_0}{q} - \frac{1}{P_{OUT}}\right) - \alpha_T\right]\right] \right]$$

an implicit expression that, for high values of $P_{OUT}$ quickly approaches eq.(1), through the definition of $I_{th}$ in eq.(21).

An approximated form of eq.22 considers $\Omega_v >> \alpha_T$ allows to solve for $P_{OUT}$ (which also implies $I_{th}=I_{th0}$):

$$P_{OUT} = \frac{hV_0}{q} \alpha_T \left[ I - I_{th0} \left(\frac{\Omega_v + \alpha_T}{\Omega_v} \left[1 + \frac{\Omega_v + \alpha_T}{\Omega_v} \left(\frac{I_{ph}hV_0}{q} - \frac{1}{P_{OUT}}\right) - \alpha_T\right]\right] \right]$$

Eq.(22a) displays the continuous increase of $P_{OUT}$ with $I$ and puts into evidence the mathematical role of $I_{th}$: to sharpen the transition across the threshold range. Low values of $I_{th}$ make a sharper transition than high ones.

IV. EXPERIMENTAL RESULTS

Fig. 2 shows the original experimental curves, that prompted the study of the DC model, superimposed to the calculated characteristics. The calibration only needed to measure two values of $I_{th}$ at two different controlled total losses $\alpha_T$ to obtain the important parameters $\Omega_v$ and $I_{th0}$ from which, given the operating frequency $V_0$, the saturation current $I_{ph}$ of the Shockley component has been calculated. Any of the experimental curves then allows to find the total efficiency (including the coupling factor that depends on the measurement apparatus), which is all is needed to predict the DC behaviour upon different values of the total losses.

CONCLUSIONS

The proposed model brings more insight into the properties and performances of a laser diode than reported in the present paper. The spectrum itself could be studied, as well as the multimodal response, upon proper evaluation of the reflection function that modulates the cavity time. Concepts as gain, gain clamp and optical transparency may be investigated under a more device-oriented point of view.

In any case, the basic equation of the DC characteristics is now available upon the simple measurement of a couple of values, from the very standard data that are usually collected during laser testing and design.

REFERENCES