Process Modelling and Simulation
with Application to the MATLAB/Simulink Environment

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COUNTRIES & REGIONS...

Italy / Sardinia / Cagliari
(60M / 1.6M / 154k )

Czech Republic / Zlín Region / Zlín
(10.5M / 600k / 75k)
Universities:

**Università di Cagliari**
(more than 30,000 students)
- Faculty of Biology and Pharmacy
- Faculty of Engineering and Architecture
- Faculty of Medicine and Surgery
- Faculty of Sciences
- Faculty of Economic Sciences, Law and Political Sciences
- Faculty of Humanities

**Tomas Bata University in Zlín**
(more than 10,000 students)
- Faculty of Technology
- Faculty of Management and Economics
- Faculty of Multimedia Communications
- Faculty of Applied Informatics
- Faculty of Humanities
- Faculty of Logistics and Crisis Management

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Faculty of Applied Informatics

Studies:
- Bachelor’s Degree studies
- Follow-on Master’s Degree studies
- Ph.D. Degree studies
- A number of courses in English for International students!

R&D in:
- Applied Informatics
- Security Technologies
- Automatic Control
- Measurement and Instrumentation

🤩 You are Welcome! 😊

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1. **MOTIVATION – WHY?**

- 😊 it saves time!
  (slow real-time experiments…)

- 😊 it saves costs!
  (expensive real-time experiments, repairs…)

- 😊 it saves health & lives!
  (hazardous real-time experiments, dangerous conditions…)

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2. **APPROACHES – HOW?**

- **ANALYTICAL** approach:
  - material & energy balances
  - mathematical description of physical, chemical, biological sub-processes…

  **analytical (internal, state-space) model**

  - structure & behaviour model
    (model variables & parameters correspond to real process variables & parameters)

  - valid in wide range of input variables and modes
    (often not allowable under real conditions…)

  - knowledge of the process + mathematics, physics, chemistry, biology…

  - design stage of a new technology
    (real plant still does not exists…)
2. **APPROACHES – HOW?**

- **EMPIRICAL approach:**

  - real-time measurements (input → output)
  - measured data processing & evaluation
    (model structure choice, identification, …)

  experimental (external, input-output) model

  - only behaviour model
    (model variables & parameters **DO NOT** correspond to real process variables & parameters)
  - process must already exist
    (real-time measurements)

  - usually simpler model
  - often more accurate model
    (for the measured range of input signals!)
  - usually time-demanding…

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2. **APPROACHES – HOW?**

**COMBINATION**
Analytical + empirical
(basic model structure by an analysis + parameters via experiments…)

.mathematical model = SIMPLIFIED reality
(some sub-processes unknown, some neglected…)

...what to model...?

TRADE-OFF
(model accuracy x model complexity)

...how accurate model is needed...?

...what to neglect...?

...how complex the model can be...?
2. **APPROACHES – HOW?**

- schematic picture
- definition of variables (input, output, state)
- simplifying assumptions
- energy / material balances
- steady-states analysis
- choice / estimate / determination of model parameters
- process variables limits, model validity
- choice of initial / boundary conditions & operating point(s) for simulation
- implementation of the model
- simulation experiments
- experiments evaluation
- model verification / corrections…
- …
3. ILLUSTRATIVE EXAMPLES

ROOM heating process:

- schematic picture:

- definition of variables:
  
  *inputs*: - heat power $P(t)$ in [W]
  
  - outdoor temperature $T_c(t)$ in [K]

  *states*: - room temperature $T(t)$ in [K]

  *outputs*: - room temperature $T(t)$ in [K]

- simplifying assumptions:
  
  - ideal air mixing
  
  - constant process parameters
    (air volume $V$, density $\rho$, heat capacity $c_p$, overall heat transfer coefficient $\alpha$, heat transfer surface area $A$, …)

  - heat accumulation in the walls neglected

  - …
3. ILLUSTRATIVE EXAMPLES

- **ROOM heating process:**
  
  - **Energy / material balances:**
    - \( P(t) = \alpha A \left( T(t) - T_C(t) \right) + \frac{d}{dt} \left[ V \rho c_p T(t) \right] \)
    
    (heat loss due to exchange of air and heat conduction through the walls)
  
  - **Steady-states analysis:**
    
    \( \frac{d}{dt} = 0 \Rightarrow P^s = \alpha A \left( T^s - T_C^s \right) \)

\[
\begin{align*}
\frac{dT(t)}{dt} &= \frac{1}{V \rho c_p} P(t) - \frac{\alpha A}{V \rho c_p} \left[ T(t) - T_C(t) \right]; \quad T(0) = T^s \\
T^s &= T_C^s + \frac{P^s}{\alpha A} \\
\alpha &= \frac{P^s}{A(T^s - T_C^s)}
\end{align*}
\]
3. ILLUSTRATIVE EXAMPLES

- ROOM heating process:

-choice / estimate / determination of model parameters:

  - \( A = 55 \, [m^2], \, V = 70 \, [m^3] \) …measured
  - \( \alpha = 1.82 \, [W/(m^2K)] \) …estimated (steady-state model for \( P^s = 2000 \, [W] \) and \( \Delta T^s = 20 \, [K] \))
  - \( \rho = 1.205 \, [kg/m^3], \, c_p = 1005 \, [J/(kgK)] \) …taken from the literature (for \( T = 20 \, [^°C] \))

-process variables limits, model validity

  - no singular states…
  - model valid in common (reasonable chosen) conditions…

-choice of initial / boundary conditions & operating point(s) for simulation

  - \( T(0) = 25 \, [^°C] = 298.15 \, [K] \)
  - \( P = 2000 \, [W] \)
  - \( Tc = 5 \, [^°C] = 278.15 \, [K] \)
3. **ILLUSTRATIVE EXAMPLES**

- **ROOM heating process:**
  - implementation of the model (steady-state model, dynamic model…)
  - simulation experiments
  - experiments evaluation
  - model verification / corrections…
  
  $T^s = T_C^s + \frac{P^s}{\alpha A}$
  
  $\frac{dT(t)}{dt} = \frac{1}{V \rho c_p} P(t) - \frac{\alpha A}{V \rho c_p} [T(t) - T_C(t)]; \quad T(0) = T^s$
  
- linear 1\textsuperscript{st} order system,
- with lumped parameters,
- continuous-time,
- deterministic,
- multivariable (MIMO),
- time-invariant…
4. **INTRODUCTION TO MATLAB & SIMULINK**

- basics of MATLAB programming (displaying process static characteristics)
- basics of Simulink (solving differential equations – process dynamics responses)
- building Simulink blocks, masking...
ILLUSTRATIVE EXAMPLES

Mass-spring-damper system:
(simplified car shock absorber)

schematic picture:

definition of variables:

**inputs:** - forcing function $F(t)$ in [N]

**states:** - position $y(t)$ in [m]
  - velocity $v(t) = \frac{dy(t)}{dt}$ in [m/s]

**outputs:** - position $y(t)$ in [m]

simplifying assumptions:

- **ideal spring**
- **well lubricated, sliding surface** (wall friction modelled as viscous damper)
- constant process parameters (spring constant $k$, friction constant $b$, mass of load $m$, ...)
- …
ILLUSTRATIVE EXAMPLES

- **Mass-spring-damper system:**

  ...performing a force balance:
  (& utilizing Newton’s 2\textsuperscript{nd} law of motion...)

  \[ \Sigma \text{forces} = ma(t) = m \frac{d^2 y(t)}{dt^2} \]

  \[ F(t) - F_k(t) - F_b(t) = m \frac{d^2 y(t)}{dt^2} \]

  spring force \quad friction force (viscous damper)

  \[ [N] + \left[ \frac{N}{s} \right] m \cdot \left[ \frac{N}{m} \right] = [N] \]

  \[ m y''(t) + b y'(t) + k y(t) = F(t) ; \quad y(0) = \ldots ; y'(0) = \ldots \]

  steady-states analysis:

  \[ \frac{d}{dt} = 0 \]

  \[ k y^S = F^S \]

  \[ y^S = \frac{1}{k} F^S \]

  \[ k = \frac{F^S}{y^S} \]
ILLUSTRATIVE EXAMPLES

Mass-spring-damper system:

- choice / estimate / determination of model parameters:
  
  \[
  m = 500 \text{ [kg]} \quad \text{load of mass (measured)}
  \]
  
  \[
  k = 2000 \text{ [N/m]} \quad \text{spring constant (taken from the literature)}
  \]
  
  \[
  b = 400 \text{ [N/(m/s)]} \quad \text{friction constant / viscous damping coefficient (taken from the literature)}
  \]

- process variables limits, model validity

  - no singular states…
  
  - model valid in common (reasonable chosen) conditions…

- choice of initial / boundary conditions & operating point(s) for simulation

  \[
  y(0) = dy(0)/dt = 0 \text{ [m]}
  \]

  \[
  F = 50 \text{ [N]}
  \]
ILLUSTRATIVE EXAMPLES

- **Mass-spring-damper** system:
  - implementation of the model (steady-state model, dynamic model...)
  - simulation experiments
  - experiments evaluation
  - model verification / corrections...

\[
y^s = \frac{1}{k} F^s
\]

\[
my''(t) + by'(t) + ky(t) = F(t); \quad y(0) = y'(0) = 0
\]

- linear 2\textsuperscript{nd} order system,
- with lumped parameters,
- continuous-time,
- deterministic,
- single-input single-output (SISO),
- time-invariant...
ILLUSTRATIVE EXAMPLES

- **Mass-spring-damper** system:

  - **Transfer function** description (using the Laplace transform):

    \[
    m s^2 Y(s) + b s Y(s) + k Y(s) = F(s)
    \]
    \[
    Y(s) \left( m s^2 + b s + k \right) = F(s)
    \]
    \[
    Y(s) = \frac{1}{m s^2 + b s + k} = G(s)
    \]
    \[
    \frac{Y(s)}{F(s)} = \frac{1}{m s^2 + b s + k} = G(s)
    \]

  - **Gain** “K”
  - **Damping coefficient** “ξ”
  - **Time-constant** “T”
  - **Natural frequency**

  - **Steady-state gain**:
    \[
    K = \frac{1}{k} = \frac{1}{2000} = 0.0005 \text{ [N/m]}
    \]

  - **Time-constant** (natural period / inverse natural frequency):
    \[
    T = \frac{\sqrt{m}}{\sqrt{\frac{k}{2000}}} = \sqrt{\frac{500}{2000}} = 0.5 \text{ [s]}
    \]

  - **Damping coefficient**:
    \[
    \xi = \frac{b}{\sqrt{4km}} = \frac{400}{\sqrt{4 \times 10^6}} = 0.2
    \]
ILLUSTRATIVE EXAMPLES

- **Mass-spring-damper system:**

  - **state-space description:**
  - Define states as: \( x_1(t) = y(t) \) ... and input as: \( u(t) = F(t) \) ... then output will be: \( y(t) = x_1(t) \)
  - \( x_2(t) = \frac{dy(t)}{dt} \)

  - Then it holds: \( \frac{dx_1(t)}{dt} = \frac{dy(t)}{dt} = x_2(t) \) ... and the 2\(^{nd}\) order model can be rewritten into two 1\(^{st}\) order DE:
    - \( \frac{dx_1(t)}{dt} = x_2(t) \)
    - \( \frac{dx_2(t)}{dt} = -\frac{k}{m} x_1(t) - \frac{b}{m} x_2(t) + \frac{1}{m} u(t) \)

  - In the matrix form:
    \[
    \begin{bmatrix}
    x_1'(t) \\
    x_2'(t)
    \end{bmatrix} =
    \begin{bmatrix}
    0 & 1 \\
    -\frac{k}{m} & -\frac{b}{m}
    \end{bmatrix}
    \begin{bmatrix}
    x_1(t) \\
    x_2(t)
    \end{bmatrix}
    +
    \begin{bmatrix}
    0 \\
    \frac{1}{m}
    \end{bmatrix}
    u(t)
    \]
    \[
    y(t) =
    \begin{bmatrix}
    1 & 0
    \end{bmatrix}
    \begin{bmatrix}
    x_1(t) \\
    x_2(t)
    \end{bmatrix} +
    \begin{bmatrix}
    0
    \end{bmatrix}
    u(t)
    \]
ILLUSTRATIVE EXAMPLES

- **Mass-spring-damper** system:

  - **state-space** description:
  
  - in the compact form: \[
  \frac{dx(t)}{dt} = Ax(t) + Bu(t) \]
  
  \[
y(t) = Cx(t) + Du(t)
  \]

  - and initial conditions: \[
  x(0) = [x_1(0) \quad x_2(0)]^T = [0 \quad 0]^T
  \]

  - ...with matrices as:

  \[
  \begin{bmatrix}
  A & B \\
  C & D
  \end{bmatrix} = \begin{bmatrix}
  0 & 1 & 0 \\
  -\frac{k}{m} & -\frac{b}{m} & 1 \\
  \frac{1}{m} & 0 & 0
  \end{bmatrix}
  \]

  ...easy simulation in the MATLAB/Simulink using the "Transfer Fcn" or "State-Space" blocks... 😊
5. **FURTHER... for control engineers...**

- linearization in a chosen operating point (for nonlinear models...)
- deviation variables & proper scaling...
- system analysis...
  - state-space / transfer function description...?
  - controllability & observability...?
  - system degree & type (P/I/D)...?
  - system gain & time-constants...?
  - (un)stable...?
  - (a)periodic response...?
  - (non-)minimum-phase behaviour...?
  - with(out) time-delay...?
- linear / nonlinear...?
- with lumped / distributed parameters...?
- continuous / discrete-time...?
- deterministic / stochastic...?
- single-variable / multi-variable...?
- time-invariant / variant...?
5. **FURTHER…for self-study…**


**Thank you for your attention!** 😊

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