Contact force regulation in wire-actuated pantographs via variable structure control and frequency-domain techniques

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One of the main problems in high-speed-train transportation systems is related to the current collection quality, that can dramatically decrease because of oscillations of the pantograph-catenary system. This problem has been addressed by means of active pantographs. In this paper we present some results about the possible implementation of variable structure control (VSC) techniques on a wire actuated symmetric pantograph. Such an actuator was suggested in the literature as a viable solution to building an active pantograph by modifying a passive pantograph currently used by Italian Railways. The use of VSC with sliding modes was considered in order to cope with the system uncertainties due to the overhead suspended catenary. Recent results about the frequency-based analysis of VSC systems featuring second-order sliding modes are exploited to avoid the performance-destroying effect of the resonant wire actuator and to get a continuous control force without using observers. We show by simulations that the contact force results are very close to the desired set-point also in the presence of measurement noise.

Keywords: active pantographs; describing function analysis; high-speed trains

1. Introduction

The regulation of the contact force between the pantograph and the overhead supply line (the catenary) has been recognised as one of the main problems to overcome in order to speed up the velocity in high-speed railways systems (Gostling and Hobbs 1983; Manabe 1992; Galeotti, Galanti, Magrini and Toni 1993; Diana, Fossati and Resta 1998; Poetsch et al. 1998).

In fact during the train run, the pantograph deforms the catenary, exerting a moving force on the strengthened rope; this fact induces oscillations which can become very large as the resonant modes are excited increasing the train speed. Resonant modes are excited by the variations of the catenary characteristics along each span due to both towers and droppers (Collina, Facchinetti, Fossati and Resta 2005).

Taking into account usual pantograph and catenary parameters, resonant modes are excited corresponding to train speed larger than 200 km/h, and this can limit the railway system performance.

It is commonly recognised that active pantographs can represent a viable solution to the contact force regulation problem (Galeotti et al. 1993; Makino, Yoshida, Seto and Makino 1997; O’Connor, Eppinger, Seering and Wormley 1997; Wu and Brennan 1997; Diana et al. 1998; Allotta, Papi, Pugi, Toni and Violi 2000; Balestrino, Bruno, Landi and Sani 2000; Pisano and Usai 2004; Collina et al. 2005). An effective controller design usually requires some relatively accurate model of the pantograph-catenary system (Eppinger, O’Connor, Seering and Wormley 1988; O’Connor et al. 1997; Wu and Brennan 1997).

Most authors represented the pantograph as a linear system (O’Connor et al. 1997; Allotta et al. 2000; Balestrino et al. 2000; Collina et al. 2005), and the distributed parameter system describing the overhead line is also represented by a linear system with lumped parameters which vary during the train run. Nevertheless, more sophisticated models of the pantograph and catenary systems have been also suggested, either for simulation or control design purposes (Eppinger et al. 1988; Poetsch et al. 1998; Collina et al. 2005). The system uncertainties suggest the use of robust control techniques (Makino et al. 1997; Wu and Brennan 1997; Allotta et al. 2000).

The actuator location is another problem to consider. The lower frame control implies the roof location for the actuator (Papi, Rinchi, Rindi and Toni 1998) and almost no limitations for the actuator size and weight, but suffers from some limitations in performance (Collina et al. 2005). Conversely the upper-frame control implies the need to counteract...
the actuator size and weight as the actuator is usually located on the pantograph (Althammer and Baldauf 1999), even if it makes easier to counteract the high frequency vibrations due to the droppers (Collina et al. 2005).

A sensible compromise could be the use of wire actuators (Allotta et al. 2000; Balestrino et al. 2000), which can be located on the train roof while exerting their control action on the upper frame by means of an iron wire of proper length.

In this paper we consider the equivalent model of the catenary as an uncertainty to compensate for by means of variable structure control (VSC) systems with sliding modes (SM) (Bartolini, Pisano, Punta and Usai 2003; Pisano and Usai 2004), and analyse the differences arising from the upper-frame or lower-frame control approaches.

With reference to a linear time invariant model of the pantograph (Allotta et al. 2000) and under the assumption that the contact force is available for measurements (Collina et al. 2005), we show in this work that a simple second-order sliding mode controller (Bartolini, Ferrara, Levant and Usai 1999) is able to track a constant reference contact-force with high accuracy in both the upper and lower frame control configurations.

Nevertheless, the accuracy dramatically decreases as soon as the wire actuator is introduced, since the chattering effect arises (Boiko, Fridman, Pisano and Usai 1999). By resorting to an approximate analysis in the frequency domain (Atherton 1975; Boiko, Fridman and Castellanos 2004) we design a dynamical pre-filter which is able to reduce the system oscillations to acceptable values (Allotta, Pisano, Pugi and Usai 2005; Boiko 2006) so that the contact force variations do not cause a significant decrease of the current collection efficiency.

In §2 we present a mathematical model of the catenary/pantograph system. In §3 we present the controller design for the two cases: (i) the pantograph actuated on the upper frame (upper-frame control, UFC); (ii) the pantograph actuated on the lower frame (lower-frame control, LFC). Section 4 presents some preliminary simulation results. In §5, we consider the use of a wire-actuator for generating the control force, an interesting solution suggested in Papi, Rinchi, Rindi and Toni (1997), well suited for practical implementation. As shown by the reported simulation results, the direct introduction of such an actuator in the proposed control systems leads to an unsatisfactory behaviour. In §6 we describe frequency-based design of pre-compensating filters devoted to alleviating the chattering, and in §7 we apply such method to the control problem under investigation. Section 8 deals with the simulation results, which show a considerable improvement of the system performance, and §9 gives some concluding remarks.

2. The system model

An exact mathematical model of the overhead suspended system, the catenary, is very difficult to define because it is a distributed system. Nevertheless, simplified models with lumped time-varying parameters have been shown to be sufficiently accurate for control system analysis and design purposes (Wu and Brennan 1997; Allotta et al. 2000; Balestrino et al. 2000), and, in particular, a linear system can approximate the pantograph dynamics in a vicinity of the working configuration (O’Connor et al. 1997; Wu and Brennan 1997; Allotta et al. 2000). Here we consider the model represented in Figure 1, which highlights the separate dynamics of the catenary and of the pantograph.

The equivalent mechanical parameters of the catenary present a periodic behaviour along each span (Balestrino et al. 2000). We consider a Fourier series expression of the equivalent parameters including the first, second and third harmonics

\[
\begin{align*}
  m_c(t) &= m_{c0} + \sum_{i=1}^{3} m_{ci} \cos \left( \frac{2\pi}{L} x(t) \right) \\
  b_c(t) &= b_{c0} + \sum_{i=1}^{3} b_{ci} \cos \left( \frac{2\pi}{L} x(t) \right) \\
  k_c(t) &= k_{c0} + \sum_{i=1}^{3} k_{ci} \cos \left( \frac{2\pi}{L} x(t) \right),
\end{align*}
\]

where \( L \) is the span length and \( x(t) \) the actual distance of the train from the closest tower.

![Figure 1. A lumped-parameters model of the pantograph-catenary system.](image-url)
By considering the system in Figure 1 the contact force has the following expression:

\[
\lambda = \max\{k_1(x_2 - x_1) + b_1(x_2 - \dot{x}_1), 0\}. \quad (2)
\]

During normal operation, the pantograph is in contact with the catenary; this implies that \( x_1 = x_c \) (see Figure 1) and that the contact force \( \lambda \) is non-negative. Therefore, the following sixth-order system derives

\[
\begin{align*}
\dot{m}_1(x_1) + b_1(x_1) + k_1(x_1) & = k_1(x_2 - x_1) + b_1(x_2 - \dot{x}_1) \\
\dot{m}_2 x_2 + b_2 x_2 + k_2 x_2 & = k_2 x_3 + b_2 x_3 + f_c(t) - k_1(x_2 - x_1) - b_1(x_2 - \dot{x}_1) \\
\dot{m}_3 x_3 + b_2 x_3 + (k_2 + k_3)x_3 & = k_2 x_2 + b_2 x_2 + f_{\delta}(t),
\end{align*}
\]

where \( f_{\delta}(t) \) is a control force acting on the lower frame and \( f_c(t) \) is a control force acting on the upper frame. Let \( z(t) = [x_1, \dot{x}_1, x_2, \dot{x}_2, x_3, \dot{x}_3]^T \) be the system state vector. The state variables \( x_1, x_2 \) and \( x_3 \) represent the deviations of the masses displacements from their equilibrium positions at rest. Then, gravity need not be included in the model as its effect is implicit in the definition of such equilibrium positions and does not affect the system dynamics.

The linear time-varying state Equations (3) are compactly expressed as follows:

\[
\begin{align*}
\dot{z}(t) & = A(t)z(t) + B_1 f_c(t) + B_2 f_{\delta}(t) \\
y(t) & = Cz(t),
\end{align*}
\]

where \( y(t) = \lambda(t) \) is the contact force and

\[
A(t) = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
-k_1 - k_1 & \frac{b_1 + b_1}{m_1(t)} & k_1 & \frac{b_1}{m_1(t)} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
\frac{k_1}{m_2} & \frac{b_1}{m_2} & 0 & \frac{k_1 + k_2}{m_2} & \frac{b_1 + b_2}{m_2} & \frac{b_2}{m_2} \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & \frac{k_2}{m_3} & \frac{b_2}{m_3} & \frac{k_2 + k_3}{m_3} & \frac{b_2 + b_3}{m_3}
\end{bmatrix},
\]

\[
B_1 = \begin{bmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & m_2 & 0 & 0
\end{bmatrix}^T, \quad B_2 = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}^T, \quad C = \begin{bmatrix}
-k_1 & -b_1 & k_1 & b_1 & 0 & 0
\end{bmatrix}.
\]

3. System analysis and controller design

3.1. I/O dynamics

The first-order input/output dynamics takes the following form:

\[
y(t) = CA(t)z(t) + CB_1 f_c(t) + CB_2 f_{\delta}(t). \quad (7)
\]

It is easy to observe that

\[
CB_1 = \frac{b_1}{m_2}, \quad CB_2 = 0
\]

Then, system (7) can be rewritten as follows:

\[
y(t) = CA(t)z(t) + \frac{b_1}{m_2} f_c(t)
\]

which means that the output variable \( y \) has a globally defined relative degree one with respect to the upper-frame control force \( f_c \).

Consider the second-order input/output dynamics by further differentiating (9), and then consider (4) into the computed derivative, which leads to

\[
\dot{y}(t) = C \begin{bmatrix}
\frac{dA(t)}{dt} + A^2(t) \\
1
\end{bmatrix} z(t) + CA(t)B_1 f_c(t) + CA(t)B_2 f_{\delta}(t).
\]

By simple computations it derives that

\[
CA(t)B_1 = \frac{1}{m_2} \left[ -\frac{b_1^2}{m_2} + k_1 - \frac{b_1(b_1 + b_2)}{m_2} \right],
\]

\[
CA(t)B_2 = \frac{b_1 b_2}{m_2m_3}. \quad (11)
\]

In practice, only one adjustable control action is employed, and the actuator location strongly impacts the output regulation control problem since the input-output relative degree is different for the two cases.

Considering (11) into (10), it yields that the output variable \( y \) has a globally defined relative degree two with respect to the control force \( f_{\delta} \).
3.2. Sliding variable design

The sliding variable is defined by the output regulation error
\[ \sigma(t) = y(t) - \lambda^*. \] (12)

The first and second derivatives of \( \sigma \) are defined by (9) and (10) taking into account that \( \lambda^* \) is a reference constant force. By (10) and (12), the second derivative of the sliding variable, \( \ddot{\sigma} = \ddot{y} \), can be explicitly evaluated,
\[ \ddot{\sigma}(t) = \ddot{y}(t) = C \left[ \frac{dA(t)}{dt} + A^2(t) \right] \xi(t) + CA(t)B_1 f_c(t) + \frac{b_1 b_2}{m_1 m_3} f_q(t) + \frac{b_1}{m_2} \dot{f}_c(t). \] (13)

3.3. Controller design

We propose a two-component control law combining a constant feed-forward ‘aiding’ term and a time-varying feedback term. Those components are denoted with the subscripts \( FF \) and \( FB \), respectively. Since it is easier to locate an actuator for the lower frame control, the feed-forward term \( f_{qFF} \) is always a component of \( f_q(t) \) such that the control forces are
\[ f_c(t) = f_{cFB}(t) \] (14)
\[ f_q(t) = f_{qFF} + f_{qFB}(t) \] (15)
with \( f_{cFB}(t) \) and \( f_{qFB}(t) \) being the two control alternatives (one of them is zero). The value of the aiding term \( f_{qFF} \), appropriately, is chosen in such a way that, with the train at rest, it can be kept, and maintained, the desired contact force value \( \lambda^* \). Since the static gain from \( f_q \) to \( y \) is 1, then it yields \( f_{qFF} = \lambda^* \).

To cope with the uncertainties affecting the system dynamics, the feedback term is defined according to the second-order sliding-mode control (2-SMC) approach (Bartolini et al. 1999). Let us introduce the “generalised sub-optimal” (G-SO) controller (Bartolini et al. 2003):
\[ u = \text{sub}(s; U, \beta) = -U \text{sign}(s - \beta s_{Mi}) \] (16)
where \( U \) is a sufficiently large constant control gain, \( \beta \) is a constant belonging to the interval \([0.5, 1]\), and \( s_{Mi} \) is the latest “singular value” of the input signal \( s \), i.e., the value of \( s \) at the most recent time instant \( t_{Mi} \) \((i = 1, 2, \ldots)\) such that \( \ddot{s}(t_{Mi}) = 0 \).

The choice of the G-SO controller needs to be motivated as it is not the unique possible one among the several different 2-SMC algorithms known in the literature (Bartolini et al. 1999). The twisting or super-twisting algorithms by A. Levant could also solve the control problem. However, we selected the G-SO since it is the unique 2-SMC algorithm that allows us to obtain a monotonical convergence transient of \( \sigma \). This is important in the considered application because a monotonical convergence of \( \sigma \) implies a monotonical convergence of \( y(t) \), thereby preventing any overshoot of the contact force. The simulation results confirm this expectation. There is also an alternative approach to solving the chattering alleviation problem in the considered application. As suggested in Levant (2003) one can increase the relative degree artificially via the combined use of high order (higher than second order) SMC and real-time differentiators of appropriate order. Nevertheless, one should expect a relatively large measurement noise in the real system scenario, and large measurement noise has potentially disruptive effects on the differentiator performance, especially when the differentiator order is large. Then the differentiator-free 2-SMC seems more appropriate for use in the actual context, and for the stabilisation of dynamics (13) the control laws (14) and (15) are considered along with the following expression for the feedback control terms \( f_{qFB} \) and \( f_{cFB} \):

**Upper frame control (UFC):**
\[ \begin{cases} 
 f_{qFB}(t) = 0 \\
 f_{cFB}(0) = 0 \\
 f_{cFB}(t) = \text{sub}(\sigma; U_c, \beta_c)
 \end{cases} \] (17)

**Lower frame control (LFC):**
\[ \begin{cases} 
 f_{qFB}(t) = \text{sub}(\sigma; U_q, \beta_q) \\
 f_{cFB}(t) = 0
 \end{cases} \] (18)

In both cases the same control algorithm is considered, but in the upper frame control the time derivative of the feedback control component \( f_{cFB} \) is considered as the auxiliary control variable. This implies that \( f_{cFB}(t) \), obtained according to (17) by time-integration of a discontinuous signal, will be a continuous function of time, while \( f_{qFB}(t) \) in (18) will be a discontinuous switching signal.

Clearly, the reason motivating this difference is the different relative degree of the sliding variable with respect to the two considered control forces. In both cases the finite time convergence of \( \sigma \) to zero is provided.

3.4. Zero-dynamics stability

Let us analyse the stability properties of the internal dynamics. The analysis is developed for the UFC control system configuration. This configuration is characterised by the input force \( f_q \) being a constant
value called $f_{qFF}$. Tedium but straightforward computations show that by defining the following non-singular change of coordinates

$$
\begin{bmatrix}
w \\
\sigma
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
-k_1 & b_1 & k_1 & b_1 & 0
\end{bmatrix}
\begin{bmatrix}
z \\
\lambda^*
\end{bmatrix}
$$

(19)

with the subset of the state vector $w \in \mathbb{R}^5$ being the internal state vector, the transformed $w - \sigma$ system dynamics is governed by the following system in normal form

$$\dot{\sigma} = CA(t)z + CBf_c(t)$$

(20)

$$\begin{bmatrix}
\dot{w}_1 \\
\dot{w}_2 \\
\dot{w}_3 \\
\dot{w}_4 \\
\dot{w}_5
\end{bmatrix} =
\begin{bmatrix}
M_0 \\
M_1 \\
M_2 \\
M_3
\end{bmatrix}
\begin{bmatrix}
w_1 \\
w_2 \\
w_3 \\
w_4 \\
w_5
\end{bmatrix} +
\begin{bmatrix}
0 \\
\frac{1}{m_1(t)} \\
0 \\
\frac{1}{m_2(t)} \\
\frac{1}{m_3(t)}
\end{bmatrix}
\begin{bmatrix}
\sigma \\
\lambda^*
\end{bmatrix}
$$

(21)

(22)

with the matrices $M_0$, $M_1$ and $M_2$ defined as follows:

$$M_0 =
\begin{bmatrix}
0 & k_c(t) & 1 \\
-k_c(t) & m_1(t) & -b_1(t) \\
-k_c(t) & m_2(t) & -b_2(t)
\end{bmatrix}
$$

$$M_1 =
\begin{bmatrix}
k_1 & 0 & 0 \\
-k_1 & m_1(t) & k_2 + k_3 \\
-k_1 & k_2 + k_3
\end{bmatrix}
$$

$$M_3 =
\begin{bmatrix}
k_1 & b_1 & 1 \\
0 & 0 & b_2 \\
0 & b_2 & b_3
\end{bmatrix}
$$

(23)

The system zero-dynamics are derived putting $\sigma = 0$ in (21)–(22). It must be proven that the corresponding $w(t)$ solutions are bounded. This is done in the Appendix.

The proposed control systems are summarised in the following statement.

**Statement 1:** Consider the pantograph model (1)–(3) with the contact force $\lambda$ as the output variable. Define the sliding variable as in (12). Then, the following propositions hold.

Upper frame control: by setting $f_q$ and $f_c$ according to (14)–(17), with sufficiently large $U_q$ and $\beta_c \in [0.5, 1)$, the output error vanishes in finite time.

Lower frame control: by setting $f_q$ according to (15)–(16), (18), with sufficiently large $U_q$ and $\beta_q \in [0.5, 1)$, the output error vanishes in finite time.

**Proof:** The sliding variable dynamics (13) can be rewritten as follows for the two UFC and LFC cases:

$$\dot{z} = \varphi_q(t) + \gamma_q f_q(t)$$

(24)

with

$$\varphi_q(t) = \varphi_q(t, z(t), f_{qFF}) = C \left[ \frac{\text{d}A(t)}{\text{d}t} + A^2(t) \right] z(t) + \gamma_q f_{qFF}$$

$$\gamma_q = \frac{b_1 b_2}{m_2 m_3}$$

(25)

The above equations have the same structure, i.e. a second-order dynamics with a time-varying uncertain drift term and a constant control gain. For the stabilisation of that class of dynamics, the second-order sliding mode approach is effective without requiring any additional signal rather than $\sigma$.

Functions $\varphi_q(t)$ and $\varphi_c(t)$ are bounded in any bounded domain of $z$, then the proposed two-component control laws based on the suboptimal algorithm are able to steer the sliding variable to zero in finite time, provided that $\beta_c$ and $\beta_q$ are freely chosen within the interval $\beta \in [0.5, 1)$ and the control gains $U_q$ and $U_c$ are taken to be sufficiently large (Bartolini et al. 2003). The finite-time convergence of $\sigma$ to zero corresponds directly to the vanishing of the output error $\lambda - \lambda^*$.

**Remark 1:** Typical SMC laws, including the considered sub-optimal 2-SMC algorithm, depend on a constant gain parameter to be tuned sufficiently large to ensure the closed-loop stability. By a conservative worst-case analysis the gain parameter can be computed off-line by proper formulas involving the worst-case uncertainty bounds. In most cases, such a value computed off-line is redundantly large and practically useless, since its use leads to an overall
degradation of the control performance in actual implementation. In practice, the controller is better calibrated by progressively increasing the gain until a satisfactory closed-loop operation is observed.

4. Preliminary simulations

In this preliminary set of simulations, no specific force-actuator is considered. The simulations have been performed in the Matlab-Simulink environment by considering the mechanical parameters reported in the Appendix. The train is kept at rest for 20 seconds, during which the nominal contact force is attained, and then a constant acceleration, according to the velocity profile reported in Figure 2, is applied. Such a velocity profile is far from a real situation but it allows for verifying the system behaviour at all speeds.

Both the lower-frame control (LFC) and upper-frame control (UFC) configuration have been simulated. The feed-forward control value and suboptimal controller parameters are the same for both the LFC and UFC cases: \( f_{\text{FF}} = 100 \text{ N}; U_c = U_q = 3000; \beta_c = \beta_q = 0.5 \). The contact force set point \( \lambda^* \) is smoothly increased from zero to the desired value in about 10 seconds. The fixed-step ODE4 integration method is used, with step \( T_s = 10^{-4} \text{s} \).

In TEST 1 and TEST 2, the UFC and LFC configurations have been considered by assuming an ideal, i.e. noise-free, measurement of the contact force. In the LFC case, in order to smooth out the unacceptable discontinuity from the force profile, the sign function in the suboptimal controller is replaced by the commonly used sat-based approximation, \( \text{sign}(\sigma) \approx \text{sat}(\sigma; \varepsilon) = \sigma/(|\sigma| + \varepsilon) \), with \( \varepsilon = 1 \). The contact force and the feedback control force components are reported in Figures 3 and 4, which show a satisfactory behaviour in both cases (slightly larger control force oscillations are observed in TEST 2). The time evolutions of both control forces highlight the velocity dependent system’s mechanical resonances, in correspondence of which larger control magnitudes are required to keep constant the contact force.

Figure 5 reports the zooms on steady state for the contact force in the TEST 1 and TEST 2. Even if the obtained regulation accuracy is satisfactory in both cases, it is interesting to note that in TEST 2 (where the SAT-based smoothing was used) the accuracy is worsened near the resonant velocities, showing that the dominance of the control, and therefore the insensitivity to parameter variations, is lost.

In TEST 3 and TEST 4, the same experiments have been repeated by including a randomly-generated additive measurement noise with maximum amplitude \( 2N \). Figure 6 shows zoomed plots of the obtained contact force, which can be compared with those reported in Figure 5 to evaluate the detrimental effect of the measurement noise. The accuracy featured by the UFC configuration is higher, as it is reasonable since the upper frame is closer to the pantograph head.
5. Control implementation via wire-actuators

In Balestrino et al. (2000), the implementation of the control forces (either $f_c$ or $f_q$) by means of a wire 'pulling down' the corresponding frame was suggested. The use of such a type of actuator constrains the control action to be negative (i.e., $f_c \leq 0$, $f_q \leq 0$). Nevertheless this can be accomplished by increasing the value of the feed-forward term, $f_{qFF}$.

Note that the constant feed-forward term $f_{qFF}$ needs to be generated via a different actuator device. Indeed it is a positive force component which pushes up the upper frame, while the wire actuator can only apply negative forces to the frame to which is attached.

A schematic representation of a pantograph of the ATR90 type (Allotta et al. 2005) with the wire actuator
is shown in Figure 7. The left plot corresponds to the LFC actuation, the right plot corresponds to the UFC actuation.

The wire actuator can be well modelled by a linear second-order dynamics with a small damping and a resonance peak (Allotta et al. 2000; Balestrino et al. 2000) according to

\[
\begin{align*}
\ddot{f}_c + 2\xi_c\omega_n\dot{f}_c + \omega_n^2 f_c &= \omega_n^2 v_c(t) \\
\ddot{f}_q + 2\xi_q\omega_q\dot{f}_q + \omega_q^2 f_q &= \omega_q^2 v_q(t)
\end{align*}
\]  

(26)

where \(v_c\) and \(v_q\) are the actuator commands. Depending on the particular frame to which the wire-actuator is connected, the length of the wire is different (obviously, a longer wire is required when the upper frame is actuated). Clearly, this difference in length affects both the resonance frequency \(\omega_n\) and the damping coefficient \(\xi\), therefore the following pairs of actuator parameters are considered in the UFC and LFC cases (Allotta et al. 2000; Balestrino et al. 2000):

**Upper frame control (UFC):** \(\xi_c = 0.24, \quad \omega_n^2 = 45.03\text{Hz}\)  

(27)

**Lower frame control (LFC):** \(\xi_q = 0.17, \quad \omega_q^2 = 80.89\text{Hz}\)  

(28)

To guarantee the wire strength condition, the feed-forward pre-loading force needs to be increased. Taking into account the range of \(f_{cFB}\) and \(f_{qFB}\) in the ideal case (Figures 3 and 4), a value of 180N was considered for the UFC configuration, and a larger value of 250N for the LFC one.

The introduction of the wire actuators (26)–(28) causes a severe deterioration of the control performance. Simulation results, obtained using the same controller parameter values as those used in §4, are reported as follows. The plots in the Figures 8 and 9 refer to the UFC and LFC configurations, respectively. It is apparent that both control configurations lead to unacceptably large oscillations of the contact force.
and control forces, even at very low train velocity, with respect to the case considered above, because of the neglected actuator dynamics in the former.

6. Chattering reduction via frequency-based controller redesign

On the basis of the results of the previous section, one might conclude that in the considered application the second-order sliding mode approach is unsuitable to be used in conjunction to wire actuators due to the excessive amount of chattering.

The presence in the control loop of parasitic dynamics, neglected in the design stage, was recently shown to be one of the main causes of chattering in ‘real’ 2-SMC systems (Boiko et al. 2004, 2007).

With the plant being linear, in order to give an estimate of the amplitude and frequency of chattering in 2-SMC systems with parasitic dynamics, a graphical procedure, based on the describing function (DF) approach, was recently suggested (Boiko et al. 2004; Boiko, Fridman, Iriarte, Pisano and Usai 2006). In Boiko et al. (2006), in particular, such a procedure was also turned into a design tool, namely a method to assign prescribed amplitude and frequency of chattering via proper tuning of the suboptimal 2-SMC parameters $U$ and $\beta$. Clearly, the chattering parameters can also be changed by shaping the plant transfer function (including sensors and actuators) by a proper linear dynamic compensator (Allotta et al. 2005; Boiko 2006).

We will show in the next section that the above tools are very useful in the considered application.

The principles of the describing-function (DF) analysis for 2-SMC systems are now recalled (the reader should refer to Boiko et al. (2006) for a detailed treatment focused on the generalised suboptimal 2-SMC algorithm).

The problem of chattering analysis has been addressed making reference to the feedback control system in Figure 10, i.e. LTI dynamics with a non-linear element (namely a 2-SMC algorithm) in the feedback loop (Boiko et al. 2004, 2006, 2007).

The proposed method is a non-standard implementation of the well-known (DF) technique (Atherton 1975). It consists of a graphical procedure for the approximate evaluation of the amplitude $A_f$ and frequency $\omega_f$ of the chattering first harmonics. It is based on the following sequence of steps:

1. Compute the DF of the non-linear element. The actual non-linear element is the sub-optimal 2-SMC algorithm (16), whose DF is (Boiko et al. 2006)

$$q = q(A_f) = \frac{4U}{\pi A_f} \left[ \sqrt{1 - \beta^2} + j\beta \right] \quad (29)$$

2. Draw a plot of the DF negative reciprocal in the complex plane. The negative reciprocal of the DF (29) is

$$-\frac{1}{q} = -\frac{\pi A_f}{4U} \left( \sqrt{1 - \beta^2} - j\beta \right) \quad (30)$$

In the complex plane, the locus (30) is a straight line parametrised by the ratio $A_f/U$, backing out of the origin and forming a clockwise angle.
If the chattering parameters are harmonic functions, it makes sense to consider the nominal (or averaged) model, i.e. the catenary parameters $m_c(t), b_c(t), k_c(t)$ in the characteristic matrix (5) are replaced by their nominal mean values $m_{c0}, b_{c0}, k_{c0}$. The frequency of variation of the catenary parameters (1) depends on the train velocity. The magnitude Bode plot of the overall (pantograph plus wire-actuator) transfer function, reported in Figure 12, shows a relatively strong magnitude attenuation in the frequency range of interest, thus, such an averaging approximation may be regarded as sensible. Denote as $A_0$ the obtained nominal characteristic matrix.

For the UFC and LFC configurations, we can compute the nominal transfer functions (TFs) $G_c$ and $G_q$ between the contact force and the input variables $f_c$ and $f_q$, respectively,

$$G_c(s) = \frac{1}{s} C(sI - A_0)^{-1} B_1$$

$$G_q(s) = C(sI - A_0)^{-1} B_2$$

The wire-actuator transfer functions are

$$G_c^{wa}(s) = \frac{o_{cn}^2}{s^2 + 2\xi_c o_c s + o_{cn}^2}$$

$$G_q^{wa}(s) = \frac{o_{qn}^2}{s^2 + 2\xi_q o_q s + o_{qn}^2}$$

Let us first analyse the UFC case. Figure 12 shows the Bode plots of the pantograph nominal TF, $G_c(s)$, of the wire actuator TF, $G_c^{wa}(s)$, and of the overall pantograph TF, $G_c^{wa}(s)G_c(s)$, together.

The possible range of chattering frequencies is highlighted by the dashed rectangle (it is the frequency range, centred approximately around 300 rad/sec where the overall transfer function phase is between $-180$ and $-270$ degrees). Unfortunately, the actuator resonance lies inside the chattering frequency range,
and this can justify the observed strong performance deterioration.

We consider the introduction of a first-order compensator with time constant 0.1 s, i.e.

\[ H_c(s) = \frac{1}{1 + 0.1s}. \] \( \text{(36)} \)

Figure 13 reports the Bode plots of the uncompensated system \( G^{\text{w}}G_c \) and of the compensated dynamics \( H_cG^{\text{w}}G_c \) including the prefilter \( H_c(s) \), together. It appears from Figure 13 that the range of chattering frequencies is shifted at lower frequencies (the actuator resonance is now outside of the range), and, furthermore, attenuation of the chattering amplitude is provided since the magnitude curve of the compensated dynamics is always below the original one.

In order to filter out the high-frequency measurement noise, an additional measurement filter \( H_m(s) \) with time constant 0.001 s is inserted at the controller input

\[ H_m(s) = \frac{1}{1 + 0.001s}. \] \( \text{(37)} \)

Clearly, the measurement filter does not alter the curves in Figure 13 in the frequency range of interest, and the relevant Bode plot is not reported. The overall

![Bode Diagram](image)

**Figure 12.** Bode plot of \( G_c \), \( G^{\text{w}} \) and \( G^{\text{w}}G_c \).

![Bode Diagram](image)

**Figure 13.** Bode plot of \( G^{\text{w}}G_c \) (uncompensated) and \( H_cG^{\text{w}}G_c \) (compensated).
The proposed control system can be summarised by the block-scheme in Figure 14. As mentioned in the §5, the constant positive feed-forward force component, $f_{qFF}$, cannot be generated via the wire actuator, which can only exert negative force values. This is the reason why in Figure 14 the component $f_{qFF}$ is constructed as a separate input to the system generated, e.g., by means of an electro-hydraulic force actuator which is one of the possible design choices.

The design procedure has also been applied to the LFC case, leading to similar considerations. The compensator was chosen as follows:

$$H_q(s) = \frac{1}{1 + s}.$$  \hfill (38)

Figure 15 reports the Bode plots of the uncompensated system $G^w_q G_q$ and of the compensated dynamics $H_q G^w_q G_q$ including the prefilter $H_q(s)$. Similar to the UFC case, the introduction of the pre-compensator $H_q$ shapes the transfer function leading to a reduction of both the frequency and amplitude of chattering.

8. Simulation results

The overall control systems, including the compensators and the measurement filters, have been simulated. Let us start with the UFC case by implementing the scheme represented in Figure 14.

In TEST 5, the suboptimal controller parameters are set as $\beta_c = 0.5$ and $U_c = 3000$. In TEST 6 the $\beta_c$ parameter is increased to $\beta_c = 0.9$ in order to alleviate chattering according to the guidelines given in the previous section. In TEST 7, the effect of random measurement noise (with maximum magnitude $2N$) is analysed by using the same controller parameters as those in TEST 6.

Figure 16 shows the zoomed contact forces in the above three tests. Comparing the results of TEST 5 and TEST 6, it is apparent that the increase of $\beta$ has positive effects from the point of view of chattering reduction. TEST 7 shows the good properties of robustness against the measurement noise. Figure 17 compares the applied control forces in the three tests.

The same analyses have been performed for the LFC case as well. With the suboptimal controller parameters set at $\beta_q = 0.5$ and $U_q = 3000$, the contact force features unacceptable oscillations even at very-low train velocities. Those very unsatisfactory results are not pictured. The introduction of the dynamic pre-compensation has worsened the system performance, compared with the results shown in §5. The increase of the $\beta_q$ parameter is considered to improve the performance according to the considerations given in §6.
In particular, in TEST 8 we increased the \( \beta_q \) parameter up to \( \beta_q = 0.99 \), obtaining a dramatic improvement of the system performance. In TEST 9, the final test, a random measurement noise having maximum magnitude \( 2N \) is incorporated by using the same parameter values as those in TEST 8. Similar to the UFC case, Figures 18 and 19 compare the zoomed contact force and control force \( f_q \).

9. Conclusions

The contact force regulation problem in active train pantographs has been faced by means of a second-order sliding mode approach.

With reference to a linear time-varying pantograph/catenary model, it has been shown that such an approach is effective in both the upper and lower frame control case by measuring only the contact force and with no information about the catenary.

The resonant dynamics of a wire actuator causes the chattering effect, that can be attenuated until acceptable values by using a linear low-pass cascade filter. The filter parameters are selected on the basis of a frequency-domain analysis of the closed-loop system that considers the nominal values of the catenary mechanical characteristics.

Simulation results also show that the presence of measurement noise does not cause unacceptable degradation of the performance.

References


Appendix A: System parameters

Table A1. Mechanical parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Value</th>
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Rewrite system (21) as

$$
\begin{align*}
\dot{w}_1 &= w_2 \\
\dot{w}_2 &= -a_0(t)w_1 - a_1(t)w_2 + a_2(t)k^* 
\end{align*}
$$

and consider the positive definite and decreasent (Khalil 2002) Lyapunov function

$$
V(w_1, w_2) = \frac{1}{2}w_1^2 + \frac{1}{2a_0(t)}w_2^2 + k_Rw_1w_2, \quad 0 < k_R < \frac{1}{\sqrt{a_0}}
$$

The derivative of (B4) along the trajectories of (B3), by omitting the argument $t$, is

$$
\dot{V}(w_1, w_2) = w_1\dot{w}_1 + w_2\dot{w}_2 = 0
$$

which is negative definite.

Appendix B: Stability of internal dynamics (21)–(22)

The dynamics (21) are decoupled from (22) and can be analysed separately first. The second-order system (21) with $\sigma = 0$ represents a generalised mass-spring-damper system with time varying bounded parameters and a bounded force input. The stability analysis of such system is not easy. The time-varying system eigenvalues, $\text{spec}(M_3(t))$, do not contain sufficient or necessary information about the stability (there are known examples of systems where both the time varying eigenvalues are zero at any time and the system has an unstable solution (Hoppenstaid 1966)). The unforced version of the problem ($\lambda = 0$) is thoroughly studied in Duc et al. (2006). Here we introduce a Lyapunov function, which is a modified version of that used in Duc et al. (2006, Th. 2.1), to prove the boundedness (not asymptotical stability) of the trajectories with non-zero $\lambda^*$. Define

$$
M_3(t) = \begin{bmatrix}
-k_R a_0(t) & \frac{1}{2}k_R a_0(t) a_1(t) \\
\frac{1}{2}k_R a_0(t) a_1(t) & \frac{1}{2a_0(t)}(\dot{a}_0(t) + 2a_0(t) a_1(t) - 2k_R a_0(t)^2)
\end{bmatrix}
$$

The following condition holds concerning the positive definiteness of the time varying matrix $M_3(t)$

$$
M_3(t) > 0 \iff f(t) = \frac{k_R}{2a_0(t)}\left[\dot{a}_0(t) + 2a_0(t) a_1(t) - 2k_R a_0(t)^2\right] > 0
$$

Once the explicit form of function $f(t)$ is derived by considering (1) and (B1) into (B7), it turns out that it depends on the train position and velocity profiles. It can be studied by considering the catenary parameter values reported in the Appendix and an appropriate meaningful profile for the train position and velocity. The velocity profile was considered in Figure B1 (left), comprising acceleration, deceleration and travel at constant velocity. By selecting $k_R = 0.001$, and plotting the function $f(t)$ using the considered velocity profile, one obtains the time profile reported in Figure B1 (right). In light of (B7), the positive definiteness of $M_3(t)$ implies that the trajectories of system (B3)
are bounded. Once the boundedness of $w_1$ and $w_2$, is proven the analysis of system (22) becomes trivial since the matrix $M_f$ is Hurwitz, which means that system (22) can be viewed as a bounded-input-bounded-state (BIBS) LTI system with bounded input terms. The analysis of the LFC case is similar and it is omitted for brevity. We remark that the validity of the proposed stability result assumes the specific parameter values given in the Table 1 and the chosen train velocity profile.

Figure B1. The velocity profile (left) and function $f(t)$ in the stability test.