A model-based sliding mode control methodology applied to the HDA-plant

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Abstract

A sliding mode control methodology using output information is demonstrated in application to the HDA-plant, a plant for production of benzene. This process is a highly integrated, non-linear large scale process with non-minimum phase and relative degree zero characteristics. The non-linear control law is designed on the basis of a linear observer-based control system. The non-linear control law uses the states of the linear observer. The performance in the sliding mode is determined by a linear stable sub-manifold of the linear closed loop control system chosen via a robust pole selection scheme. The sliding mode control is optimized to operate in a wide operating region.

Keywords: Sliding-mode control; $H_\infty$-Control; Chemical processes

1. Introduction

The HDA-plant, a chemical plant for production of benzene, has been of longstanding interest to chemical process engineers [1–3] and control engineers [4–6]. Since this non-linear, large scale and relative degree zero process is highly integrated and non-minimum phase, it forms an important test bed for new control approaches. The HDA-process model was initially implemented as a 68 state model by Brognaux [4] and then extensively re-developed into a 270-state system by Cao et al. [7] under SpeedUp. The model has been re-implemented under Aspen Custom Modeler1 by the authors for this paper. The plant has undergone a detailed analysis with respect to the available combinations of single-input–single-output (SISO) control schemes and the most effective actuators [5,8,9]. However, control schemes have been generally limited to SISO-control methods as seen in Luyben et al. [6] for a different HDA-model simulation set-up. Hence, the application of model-based, multi-variable control as considered here is novel. For the multivariable control, a linear mixed sensitivity $H_\infty$-control problem is posed. Enforcing pole separation for the linear controller and observer system, a non-linear control is augmented using the observer states. The non-linear controller improves robustness and performance via pseudo-sliding forcing the closed loop states into the vicinity of a stable manifold of the linear closed loop control.

The article discusses first the plant, problems of non-linearity, the tracking problem and the control methodology. Finally, the model reduction, controller design and simulation results are presented.

2. The HDA-plant: a benzene producing chemical process

In the HDA-plant (Fig. 1), benzene is produced via hydrodealkylation (HDA) of toluene. The reactions taking place comprise an exothermic reaction and an equilibrium reaction. There are two input substances, toluene and hydrogen and three product substances, benzene, diphenyl and methane (Fig. 1). For basic operation, Brognaux [4] and also Cao et al. [7] introduced PID controllers to stabilize the system and to keep certain plant variables in a well-defined range. The remaining measurable output values in Table 1 have to be controlled. A tracking controller is supposed to follow production demand changes while keeping the interaction with the other four measurements as low as possible. Theoretical input/output controllability analysis applied to the HDA-plant by Cao and Rossiter [8], Cao and Rossiter [9] and Cao et al. [5] showed that six out of...
13 possible actuators are the most effective for control (Table 3). Further, the external disturbances of Table 2 affect the closed loop system performance. Chemical plants are often subject to delays: These are to be expected within the HDA-plant for the product purity measurement. Cao et al. [10] investigated delays of up to 10 min. The HDA-plant model is highly non-linear. One characteristic of the responses of output measurements of the open loop plant to small positive and negative step changes of the inputs is that they are not inverted to each other. Fig. 2 shows that the non-linear fast dynamics seem to match the responses of the Aspen-Custom-Modeller-provided linearizations, while slow dynamics prove to be highly non-linear. This has been also verified by investigating the frequency responses of the linearizations in the considered operation envelope. The next section will consider the necessary theoretical background for controller design which is employed for the HDA-plant in connection with an order reduced linear model.

3. An $H_{\infty}$-controller based approach

Suppose a linear, time-invariant system is given by:

$$\dot{x} = Ax + Bu + Bd, \quad x \in \mathbb{R}^n, \quad B \in \mathbb{R}^{n \times m}, \quad Bd \in \mathbb{R}^{n \times m_d}$$

$$y = Cx + Du + D_d d, \quad C \in \mathbb{R}^{p \times n}$$

st. sp. $\left[ \begin{bmatrix} A & B & Bd \\ C & D & D_d \end{bmatrix} \right]$.

where $u \in \mathbb{R}^m (m \geq m)$ are the available actuators, $d$ the plant disturbances and $y \in \mathbb{R}^p$ the output measurements. A mixed sensitivity $H_{\infty}$-control problem can be posed

<table>
<thead>
<tr>
<th>Measurements</th>
<th>Required range</th>
<th>Scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Flash inlet temperature</td>
<td>98–102 (F)</td>
<td>$O_1 = \frac{1}{4}$</td>
</tr>
<tr>
<td>2. Production rate</td>
<td>240–280 (lb mol/h)</td>
<td>$O_2 = \frac{1}{5}$</td>
</tr>
<tr>
<td>3. Product purity</td>
<td>At least 99.97% (mol-%)</td>
<td>$O_3 = \frac{1}{10}$</td>
</tr>
<tr>
<td>4. Hydrogen to aromatics ratio</td>
<td>4.9–5.1 (mol/mol)</td>
<td>$O_4 = \frac{1}{5}$</td>
</tr>
<tr>
<td>5. Flash outlet vapour pressure</td>
<td>458–460 (psia)</td>
<td>$O_5 = \frac{1}{9}$</td>
</tr>
</tbody>
</table>

Table 1

Measurements available for control

$^a$ Substance quantity: 1 lb mol = 453.593 mol, (lb mol) = pound-mole; Temperature: $T$ (F) = $1.8T$ (C) + 32, (F) = Fahrenheit; pressure: 1 psia = 6.895 kPa, (psia) = pounds per square inch (absolute pressure).

<table>
<thead>
<tr>
<th>Disturbance</th>
<th>Nominal value</th>
<th>Scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Pre-flash cooler: cooling effluent temperature</td>
<td>59 (F)</td>
<td>$I_1 = 9$</td>
</tr>
<tr>
<td>2. Purge down stream pressure</td>
<td>350 (psia)</td>
<td>$I_5 = 50$</td>
</tr>
</tbody>
</table>

Table 2

Disturbances influencing the control
according to Fig. 3 choosing the sensitivity weight $W_y$ as part of the controller and including integrators to achieve a steady state error of 0. Further, the actuator is weighted by $W_{uu}$ and $W_{ur}$, a filter for unstructured uncertainty and a filter limiting the derivative of the actuator signals. $W_p$ shapes the class of disturbances $d$. The $H_\infty$-optimization criterion minimizes the $H_\infty$-norm $\|T_{y,u}[\|e\|]\|$ of the transfer function $T_{y,u}[\|e\|]$ from the exogenous inputs $[e^T, r^T]^T$ to the controlled outputs $y_H = [y_H^T, y_{uH}, y_{rH}]^T$ (Fig. 3). Robustness with respect to unstructured additive uncertainty, $G + \Delta$, $\|W_{uu}K_xW_yS_\infty\|_\infty < 1$ of the nominal plant, $G \equiv (A, B, C, D)$ is achieved if $\|W_{uu}W_{yy}S_\infty\|_\infty < 1$ where $K_x$ is the designed control and $S_\infty(s) = (I - G(s)K_x(s)W_y(s))^{-1}$. A sufficient condition for this is that $\|T_{y,u}[\|e\|]\|_\infty \leq 1$. Weights $W_y$ have been used with poles at 0 which precludes the use of the standard $H_\infty$-synthesis procedure of Zhou et al. [11]. Safonov’s pole shifting methodology has thus been used [12].

### 3.1. A linear observer based closed loop control system with auxiliary non-linear control input signal

Considering only the controlled plant input $u$ and the measured output $y_H$ from Fig. 3, the plant augmented with the $H_\infty$-design weights is given by $G$:

$$\begin{bmatrix} x_h \\ y_H \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_h \\ u \end{bmatrix},$$

and a linear controller for (1) with auxiliary inputs $u_{NL}$ and $u_X$ is

$$\begin{bmatrix} \Delta x_h \\ \Delta y_K \end{bmatrix} = \begin{bmatrix} A_K & B_{KL} & B_X \\ C_K & D_K \end{bmatrix} \begin{bmatrix} \Delta x_h \\ \Delta u_{NL} \\ \Delta u_X \end{bmatrix},$$

where the controller $K_{x_h} \equiv (A_K, B_K, C_K, D_K)$ has been designed for $u_K = y_H$ and $u = y_K$ so that $\Delta x_h$ are observer states and at least a weak pole separation principle (13], p. 178) applies to this closed loop system. The matrices $B_{NL}, B_X$ and the input $u_X$ are determined later.

### Table 3

<table>
<thead>
<tr>
<th>Actuator</th>
<th>Unit</th>
<th>Scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Benzene column splitter: reflux ratio</td>
<td>(s/mol/h)</td>
<td>$Z_1 = 0.265$</td>
</tr>
<tr>
<td>2. Compressor horse power</td>
<td>(hp)</td>
<td>$Z_2 = 64.494$</td>
</tr>
<tr>
<td>3. Pre-flash cooler duty</td>
<td>(BTU/h)</td>
<td>$Z_3 = 2658.677$</td>
</tr>
<tr>
<td>4. Gas feed flow</td>
<td>(lb mol/h)</td>
<td>$Z_4 = 98.500$</td>
</tr>
<tr>
<td>5. Toluene feed flow</td>
<td>(lb mol/h)</td>
<td>$Z_5 = 55.875$</td>
</tr>
<tr>
<td>6. Purge outlet valve opening</td>
<td>-</td>
<td>$Z_6 = 9.183$</td>
</tr>
</tbody>
</table>

* Power: 1 hp = 745.71W, (hp) = horse power; energy: 1 BTU = 1054.185155 J, (BTU) = British thermal unit.
For the closed loop system investigated here, assume the auxiliary signal $u_{NL}$ (Fig. 4) is added to the linear controller output signal $y_K$, so that $u = u_{NL} + y_K, u_K = y_H$. For $e_G = x_G - \hat{x}_G$, the closed loop can be derived from a transfer matrix formula for a feedback connection ([11], p. 66)

\[
\begin{bmatrix}
\dot{x}_G \\
\dot{e}_G
\end{bmatrix} =
\begin{bmatrix}
A & A_1 \\
A_2 & A_3
\end{bmatrix}
\begin{bmatrix}
B & 0 \\
B_1 & -B_Y
\end{bmatrix}
\begin{bmatrix}
x_G \\
e_G
\end{bmatrix}
\begin{bmatrix}
u_{NL} \\
u_Y
\end{bmatrix}
\]

where $B_1 = -B_{NL} - B_KD_2R_2 + B_GR_2$ and $R_2 = (I - D_KD_G)^{-1}$. If $e_G$ defines the dynamics of an observer error with at least weak pole separation, then generally $\|A_2\| \ll \|A_3\|$ where $\|\|$ is the (induced) Euclidean norm. Hence, exact observer/controller pole separation applies if $B_{NL} \triangleq -B_KD_2R_2 + B_GR_2, B_X \triangleq A_2, u_Y \triangleq x_G$ and the matrix $A_3 + A_2$ has stable eigenvalues. It is readily understood that the exogenous inputs $[\xi', r']$ do not affect the actual closed loop stability and this permits them to be neglected when considering stability. The introduction of integral action ensures zero steady state error for a given demand $r = \text{const}$. Having established a stable linear closed loop system with pole separation of observer and feedback, a sliding-mode state-feedback control can be constructed.

3.2. Derivation of a sliding mode hyperplane from a stable plant representation

Sliding-mode control is a powerful, non-linear, robust control method which has been successfully applied to many practical systems such as furnaces [14], car engines [15,16] or induction motor driven tram systems ([16], p. 271). The approach of this non-linear control method is to achieve a sliding motion by forcing the closed loop states onto a stable sub-manifold of the state-space, the sliding-mode hyperplane. By assuming that uncertainty, un-modeled non-linearities and disturbances are matched, i.e. confined to the range of the actuator, a controller achieving a sliding motion renders the closed loop system invariant to these disturbances and introduces reduced order sliding dynamics. For linear uncertain systems, an extensive framework of non-linear controller structures has been developed [16,17]. The most common choice of a sliding mode surface is a linear hyperplane [16]. Furthermore, problems of unmatched uncertainty can be suitably addressed for linear uncertain systems [17]. The design of a sliding surface and a non-linear controller will be undertaken for the stable system pair $(A, B)$ in (2), using the linear observer states and an appropriate non-linear auxiliary input $u_{NL}$. 

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**Fig. 3.** Augmented plant for $H_\infty$-optimization considering additive uncertainty $\Delta - W_{uu}, (\|\Delta\|_\infty \leq 1)$.

**Fig. 4.** Closed loop system with auxiliary inputs.
Assuming \( B \) is full column rank, then a non-singular linear transformation \([17]\) exists so that, disregarding the interaction with the observer error and the exogenous input, \((A, B)\) can be represented as:

\[
\dot{x} = Ax + Bu_N, \quad x \in \mathbb{R}^N, \quad A \in \mathbb{R}^{N \times N}, \quad B \in \mathbb{R}^{N \times m},
\]

\[
B = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}, \quad B_2 \in \mathbb{R}^{m \times m}, \quad u_N \in \mathbb{R}^m.
\]

(3)

where \( B_2 \) is full rank, \( A \) a stable matrix. Define \( A \) to be a diagonal matrix containing the eigenvalues of \( \mathcal{A} \):

\[
\mathcal{AV} = \mathcal{V} \Lambda, \quad \Lambda \in \mathbb{C}^{N \times N}.
\]

(4)

The following Lemma, a theoretical extension of an approach of Bhatti ([15], pp. 158–159), shows that it is possible to find a transformation realizing the canonical form required for sliding mode control.

**Lemma 1.** Suppose there exists a choice of \((N-m)\) distinct eigenvalues of \( A \), which contains \( r, 0 \leq r \leq N - m \), real eigenvalues \( \lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_r \in \mathbb{R} \):

\[
\mathcal{A}[v_1 \ldots v_r] = [v_1 \ldots v_2] \operatorname{diag}(\lambda_1, \ldots, \lambda_r)
\]

and \( \frac{N-r}{2} \) pairs of complex conjugate eigenvalues \( \lambda_{r+1} = \lambda_{r+2} \ldots, \lambda_{N-m-1} = \lambda_{N-m} \in \mathbb{C} \setminus \mathbb{R} \):

\[
\mathcal{A}V_C = V_C \operatorname{diag}(\lambda_{r+1}, \lambda_{r+2}, \ldots, \lambda_{N-m}).
\]

with

\[
V_C \overset{\text{def}}{=} [v_{r+1} + iv_{r+2} v_{r+1} - iv_{r+2} \ldots]. \quad v_k \in \mathbb{R}^N, \quad 1 \leq k \leq N - m.
\]

Then there is a transformation matrix \( T \) and a matrix \( \Sigma \) with eigenvalues \( \lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{N-m} \) such that

\[
T = \begin{bmatrix} I & 0 \\ S_1 & S_2 \end{bmatrix},
\]

\[
TAT^{-1} = \begin{bmatrix} \Sigma & A_{12} \\ 0 & \Omega \end{bmatrix},
\]

(5)

\[
S_2 \in \mathbb{R}^{m \times m}, \quad \Omega \in \mathbb{R}^{m \times m},
\]

where

\[
[S_1 \quad S_2] = [0 \quad I_m] U^T, \quad \dot{V} = U \tilde{\Sigma} V,
\]

\[
\tilde{\Sigma} = \begin{bmatrix} \tilde{\Sigma}_1 \end{bmatrix}, \quad \tilde{V} \overset{\text{def}}{=} [v_1 \ldots v_r v_{r+1} \ldots v_{N-m}]
\]

and \( U \in \mathbb{R}^{N \times N}, \quad V \in \mathbb{R}^{(N-m) \times (N-m)} \) orthogonal and \( \tilde{\Sigma}_1 > 0 \), \( \tilde{\Sigma}_1 \in \mathbb{R}^{(N-m) \times (N-m)} \) diagonal resulting from a singular value decomposition, if and only if \([I_{N-m}] V\) has full rank.

**Proof.** For brevity, the proof is sketched. In Herrmann [18], the full proof is provided. Suppose

\[
U = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix}, \quad U_{11} \in \mathbb{R}^{(N-m) \times (N-m)}
\]

then \( S_1 = U_{12}^T, \quad S_2 = U_{22}^T, \quad [I_{N-m}] V = U_{11} \tilde{\Sigma}_1 V \). Hence, \( T \) exists only if \( U_{12}^T \) or \( U_{22}^T \) is invertible and \([I_{N-m}] V\) has only full rank if \( U_{11} \) has. Schur’s formula implies that an invertible \( U_{11} \) is equivalent to an invertible \( U_{22} \). Therefore, the existence of \( T \) is equivalent to the full rank condition for \([I_{N-m}] V\). Now, the second part of the claim (5) has to be shown. Note that

\[
[S_1 \quad S_2] \tilde{V} = 0 \iff [S_1 \quad S_2] [V_C v_1 v_2 \ldots v_r] = 0.
\]

Assume without loss of generality

\[
V = [\tilde{V} \ast], \quad \Lambda = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix},
\]

\[
\Lambda_1 = \operatorname{diag}(\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{N-m})
\]

then observing the structure of \((TAT^{-1})(TV) = (TV)\Lambda\) and using the fact that \([I_{N-m}] V\) is invertible it follows that \(TAT^{-1}\) has the structure of (5) and \( \Sigma \) has the eigenvalues \( \Lambda_1 \).

Note that if the eigenvalues of \( A \) are all real and distinct then there exists at least one choice of \((N-m)\) eigenvalues so that there is a transformation matrix \( T \) and a matrix \( \Sigma \) with the eigenvalues as chosen. Thus, the stabilized system (3) can be transformed for a suitable choice of eigenvalues into:

\[
\dot{z}_i(t) = \Sigma z_i(t) + A_{12} \phi(t)
\]

\[
\phi(t) = \Omega \phi(t) + (S_2 B_2) u_N(t).
\]

where the stable matrices \( \Sigma \) and \( \Omega \) and \( A_{12} \) are defined in (5). By construction, the hyperplane defining the sliding surface is given by

\[
S = \{ [z_i] : \phi = 0 \}
\]

Suppose the sub-state \( \phi \) can be forced to remain on the sliding mode hyperplane in finite time then uncertainty and disturbances within the range space of \( B_2 \) can be completely rejected and the control system dynamics are governed by the reduced order dynamics \( z_i = \Sigma z_i \). Define a Lyapunov matrix \( P = P^T > 0, (P \in \mathbb{R}^{m \times m}) \), where \( P\Sigma + \Omega^T P = -I_m \) and suppose uncertainty and disturbances are in the range space of \( B_2 \) and are parametric or constant bounded, then the non-linear control signal

\[
u_N = - \frac{(S_2 B_2)^{-1} P \phi}{\|P\phi\| + \delta_N \gamma_1 \left\| \left[ x_i^T \phi^T \right] + \gamma_2 \right\|},
\]

(6)

\[
\delta_N > 0, \quad \gamma_1, \gamma_2 > 0
\]

can achieve sliding, \( \phi = 0 \), for \( \delta_N \) in finite time and large enough \( \gamma_1, \gamma_2 \) [17]. However in practice, the choice \( \delta_N > 0 \) is used to prevent chattering, high frequency
switching of the non-linear control ([14] and [17], p. 15). This implies that ultimately pseudo-sliding motion, \(|\phi| \leq \delta \delta_{\text{var}}\), can be assured only where \(\delta_{\text{var}} : \mathbb{R} \Rightarrow \mathbb{R}\) is an increasing continuous function satisfying \(\delta(0) = 0\). The application of an observer also reduces chattering of the control: such control structures have been used in practice by Bhatti [15] and have been reported by Utkin [16] as practically important. However, strict mathematical proofs of robust stability for this linear observer/non-linear controller set-up have not been shown so far.

A question remains in the case of the availability of several choices for the sliding mode plane as to which choice is preferable. For high order plants, the number of choices (max. \(m = (N-m)\text{dim}\)) prevents testing of each possibility via simulation. A simulation involving performance or robustness, for assessment of the sliding mode plane prior to simulation, is preferable. It has been argued by Edwards and Spurgeon [17], using ideas of robust eigenstructure assignment, if the condition number of the matrix of right hand eigenvectors \([I_{N-m}]V\) of \(\Sigma\) is minimized then the sliding mode poles become insensitive to uncertainties or perturbations of the system matrix \(A\). The minimization of the condition number provides a robustness measure for the system in sliding motion, but does not necessarily imply good performance. It is therefore better to evaluate pole combinations with condition number close to the optimum. A search algorithm from Cao and Rossiter [9] based on the branch-and-bound technique can be used. The algorithm is generic and is applicable to any combinatorial problem where \(N-m\) elements have to be chosen with a minimal cost \(J(N-m)\) from a basis set of \(N\) elements. Each selection of \(k\), \(0 \leq k \leq N-m\), elements is associated with a cost \(J(k)\) which monotonically increases with \(k\), \(J(k+1) \geq J(k)\), when adding a new element to the existing set of \(k\) elements. Hence, consider the following characteristics:

\[ J(k) = \frac{1}{k(k+1)} \sum_{i=1}^{k} \frac{1}{\sigma_i} + \frac{1}{k+1} \sum_{i=k+1}^{N-m} \frac{1}{\sigma_i} \]

Remark 1. A matrix \([\theta_1 \theta_2 \ldots \theta_{k+1}]\) of \((k+1),(1 \leq k < N-m), m \geq N\), linearly independent vectors \(\theta_i \in \mathbb{R}^{N-m}\) has exactly \(k+1\) non-zero singular values where the largest singular value is \(\sigma_1/\sigma_{k+1}\) and the smallest is \(\sigma_{k+1}/\sigma_1 > 0\). For the matrix \([\theta_1 \theta_2 \ldots \theta_k]\) of \(k\) vectors the largest singular value is \(\sigma_{1/k} > 0\) and the smallest is \(\sigma_{k/k} > 0\). For \((k+1) = (N-m)\) the relation \(\sigma_{1/k} > 0\) is the condition number of a square matrix.

Thus, the monotonicity of \(J(k)=\sigma_1/\sigma_2\) in \(k\) can be easily used with the set of \(n\) available vectors \([I_{N-m}]V\) with \(V\) from (4). However, the algorithm has to consider that a selection of vectors \(V\) containing a complex valued vector has to include the complex conjugate eigenvector. The branch-and-bound algorithm allows this modification and it has been used for design described in the next section.

4. Controller design for the HDA-plant

The control design takes the two disturbances, six actuators and the five output measurements into account. Thus, a model linearization \(G_{HDA}(s)\) of 270 states, eight inputs and five outputs for the operation point of the nominal values of Tables 1 and 2 (middle of required value range) and a production rate of 265 lb mol/h is considered. For model order reduction and controller design, input and output scalings \(I_w = \text{diag}(I_1, I_2, \ldots, I_8)\), \(O_w = \text{diag}(O_1, O_2, \ldots, O_3)\) (Tables 1–3) have been used. From an evaluation of the Hankel singular values of the balanced realization of \(O_w G_{HDA}(jw) I_w\), a 10–20th order model was seen to be most appropriate. Balanced truncation has been preferred to achieve good high frequency model matching because of the uncertainty associated with the linearization at low frequency. A 17th order model \(\hat{G}_{HDA}\) has been obtained.

The \(H_\infty\) controller from Section 3 employs the derived order reduced model and the weights:

\[
W_u(s) = \text{diag}(1.9101, 2.3201, 0.1101, 0.1011, 0.0101, 0.0101, 0.0101, 0.0101, 0.0101, 0.0101),
\]

\[
o_o = \frac{100000}{s + 100000}
\]

\[
W_w(s) = \text{diag}(0.0077601, 0.0097901, 0.0097901, 0.0101501, 0.0101501, 0.0101501, 0.0101501, 0.0101501, 0.0101501, 0.0101501),
\]

\[
o_u = \frac{450.9s^3 + 4.813 \cdot 10^5 s^2 + 3.035 \cdot 10^8 s + 5.71 \cdot 10^9}{s^3 + 2.228 \cdot 10^5 s^2 + 1.768 \cdot 10^8 s + 2.161 \cdot 10^7},
\]

\[
W_r(s) = \text{diag}(0.1111 s + 2.143 \cdot 10^8, 0.7143s + 2.069, 1.1111s + 2, 0.9091s + 4.444, s + 1.444),
\]

\[
W_p(s) = \text{diag}(0.5, 0.01, 0.5, 0.01, 0.5, 0.01, 0.5, 0.01, 0.5, 0.01).
\]
model linearizations $G_j$ are obtained to evaluate the degree of uncertainty. This procedure keeps the set-points in reasonable proximity of the nominal set-point and does not introduce an unnecessarily high, spurious uncertainty which is introduced by testing all available combinations. An envelope of $\max_j \| G_j - \tilde{G}_{HDA} \|$ has been fitted with a transfer function. The resulting transfer function was used to construct $W_{uu}$. Safonov's pole shifting methodology [12] is used for the $H_\infty$-design, shifting the augmented plant poles by a value of $\tilde{p} = 0.0001$ for design. The choice of plant model order and the weights results in a control system order of 48 states and an additional five states for the sensitivity weight $W_s$ and integrators which are included in the closed loop. The numerical $H_\infty$-optimization with the chosen design weights gives a closed loop norm of $\| T_{y\nu}(\omega) \|_\infty \approx 2.3$. For the specified controller design weights, the resulting controller has the property that $\| W_{uu} K_\infty W_s S_\infty \| \geq 1$ (Fig. 5b) for parts of the frequency range; hence, robustness to plant non-linearities has been sacrificed for improved controller performance. Since $\| W_{uu} K_\infty W_s S_\infty \|$ is constant in large parts of the considered frequency range, a further reduction of this norm by $H_\infty$-optimization is not possible without performance degradation. Thus, this trade-off leads to an indirect reduction of the effective operation area of the closed loop controller. This procedure is common in applied $H_\infty$-control (e.g. [19]) as the primary interest is to achieve practically robust performance in the operation area of interest.

The closed loop performance and robustness of the linear control for the non-linear HDA-plant is improved by the introduction of a sliding motion using the linear observer derived from the designed $H_\infty$-control. For this reason, the linear control is modified to introduce pole separation (Section 3.1). This is found to be feasible since $A_2$ from (2) is small compared to $A_3$

$$\| A_2 \| = 0.156526 \ll \| A_3 \| = 1 \cdot 10^5, \| A \| = 2.35 \cdot 10^4$$

and the matrix $(A_3 + A_2)$ is stable. The sliding mode plane is chosen so that the condition number $\| I \|_m \approx \| V \|$ is small. The pole combination with condition number value of 222.1 yields good performance. The condition number of the eigenvector matrix for some other pole choices can reach values of the order $10^{17}$ indicating poor robustness to unmatched uncertainties. It

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![Graphs](image1.png)

(a) $S_\infty$

(b) $W_{uu} K_\infty W_y S_\infty$

(c) $(I - S_\infty)$

Fig. 5. Frequency responses of $H_\infty$-controller for linearized plant model.
has been found that other ad-hoc pole selection schemes for the sliding mode were less successful. The parameters for the non-linear control element from (6) have been chosen with $\gamma_1 = 100, \gamma_2 = 0$, which allowed practically fast attainment of the pseudo-sliding mode $\|\dot{\phi}\| < 1$ while $\delta_{NL} = 1 \times 10^{-5}$ has been chosen to prevent chattering of the control and numerical inaccuracy of the simulation.

4.1. Controller performance

The linear $H_\infty$-controller was designed so that the settling times of the linear responses were less than 5 h. The non-linear simulations show fast rise times for the production rate (Fig. 6), but long settling times. This results from the fact that low frequency uncertainty is particularly high (Section 2) which only allows the designer to obtain fast initial dynamics. Interaction with the remaining four output measurements for the $H_\infty$-controller is relatively slow (Fig. 7). Systematic tests with controllers employing various design weights have shown that increasing the bandwidth of the sensitivity function $S_\infty$ of the linear closed loop $H_\infty$-controller by adjusting the weight $W_r$ does not result in a faster response with improved interaction of the non-linear simulated closed loop system while a decrease of the required bandwidth would decrease rise times and subsequently settling. This has been particularly detected for the purity measurement response. Subsequently, a sliding mode controller is designed based on the $H_\infty$-controller. This reduces the respective interaction and gives faster settling times for the four output measurements from Fig. 7, while for the production rate the tracking response characteristics are very similar to the linear control. This has been also verified employing Fig. 6. Production rate tracking response of $H_\infty$ and linear observer based sliding mode control.

Fig. 7. Tracking errors of linear observer-based control due to production demands (see Fig. 6).
output data sampled at \( \frac{1}{\min} \) and analyzed via the mean absolute error (Table 4) of the tracking errors (\( y - r \)):

\[
\hat{e}_j = \frac{1}{N_s} \sum_{i=1}^{N_s} |y_j(t_i) - r_j(t_i)|, j = 1 \ldots 5, \tag{8}
\]

The mean absolute error reduces by at least 30% up to 90% for the four interaction outputs which is a significant improvement for general plant operation and product quality. The non-linear controller also operates in a wider envelope than the \( H_1 \)-controller. The sliding mode controller remains stable for production rate demands below 240 \( \frac{\text{lb mol}}{\text{h}} \), while the \( H_\infty \)-controller becomes oscillatory for a demand of 240 \( \frac{\text{lb mol}}{\text{h}} \) and unstable below this value (Fig. 8).

The sliding mode controller and the linear \( H_\infty \)-controller are both robust to product purity measurement delays of more than 20 min. The sliding mode control generally shows better closed loop responses to small cooling-effluent temperature disturbances than the \( H_\infty \)-controller, while for purge-down stream pressure disturbances both controllers have similar performance.

5. Conclusion

An observer based sliding-mode control scheme has been presented and applied to the highly non-linear, benzene producing HDA-plant. The plant has a particular highly non-linear steady state characteristic. A linear model is used for control design. Therefore, the linear 270-state model retrieved via a software linearization tool had to be order-reduced. The balanced truncation methodology reducing the plant model to 17 states has been found to introduce minor model errors compared to the linear 270-state model. The large low frequency gain uncertainty of the HDA-process affects the settling times of the linear control and this can be improved by introducing a non-linear sliding-mode control. The non-linear control is stable for a wide operating region and is robust to measurement delays.

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References


![Fig. 8. Non-linear responses of the two \( H_\infty \)-based controllers to a production rate ramp demand change during 0.5 and 1 h from 265 to 240 (– –, linear; —, sliding mode controller).](image)


