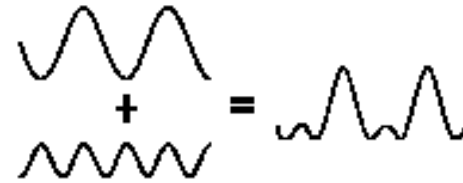
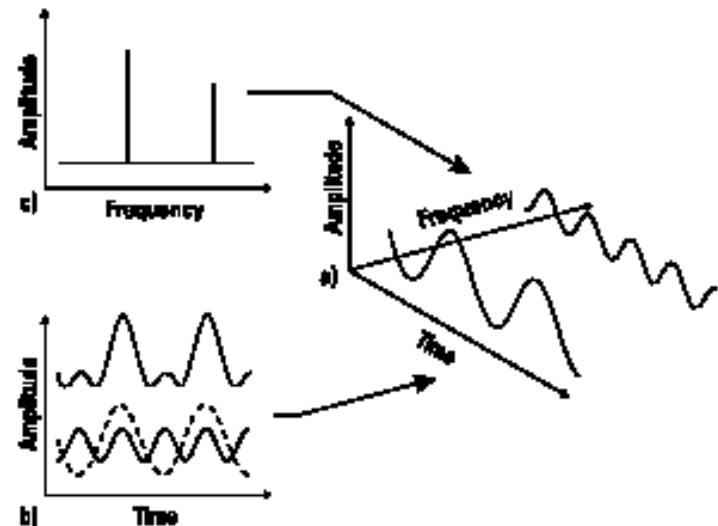


## Analisi nei domini del tempo e della frequenza

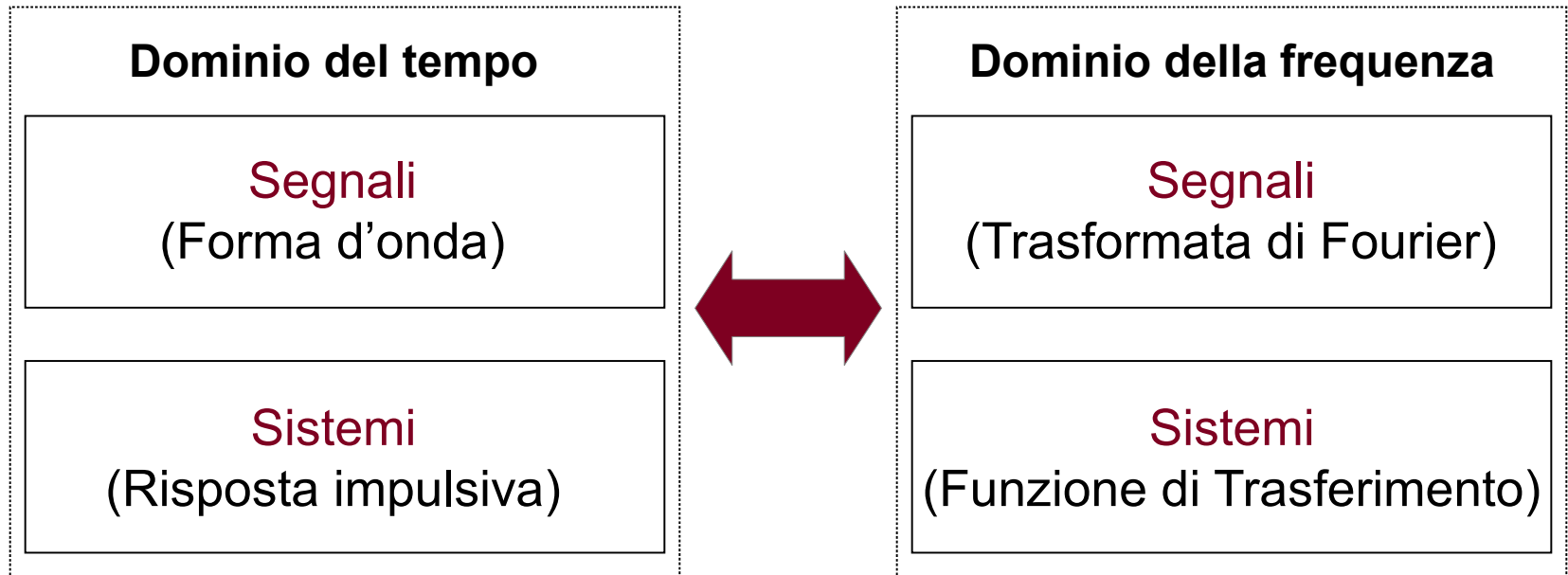
Ogni segnale reale può essere prodotto aggiungendo onde sinusoidali



- a) Coordinate tridimensionali: tempo, frequenza ed ampiezza.
- b) Vista nel dominio del tempo.
- c) Vista nel dominio della frequenza.



## Analisi nei domini del tempo e della frequenza



## Analisi nel dominio della frequenza

### Trasformata di Fourier tempo-continua

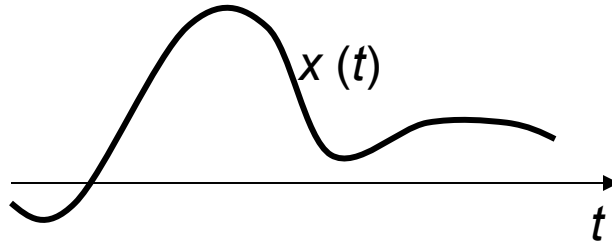
*Trasformata diretta*

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} dt$$

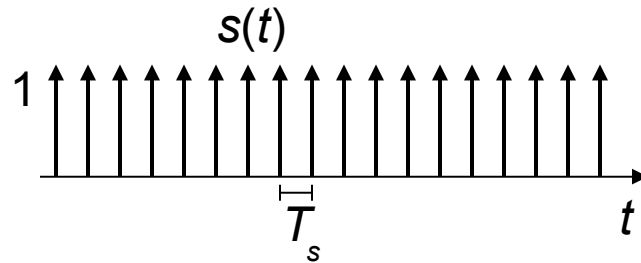
*Trasformata inversa (antitrasformata)*

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{j2\pi ft} df$$

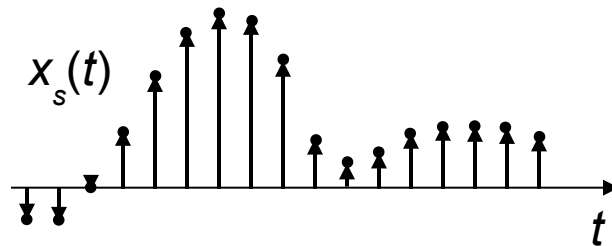
## Segnali campionati



$$x(t)$$

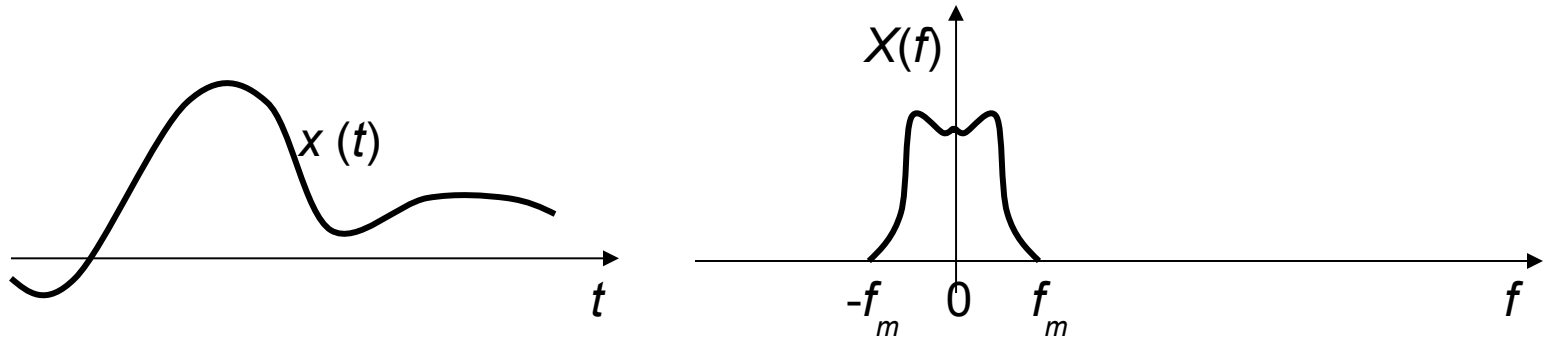


$$s(t) = \sum_{i=-\infty}^{+\infty} \delta(t - iT_s)$$

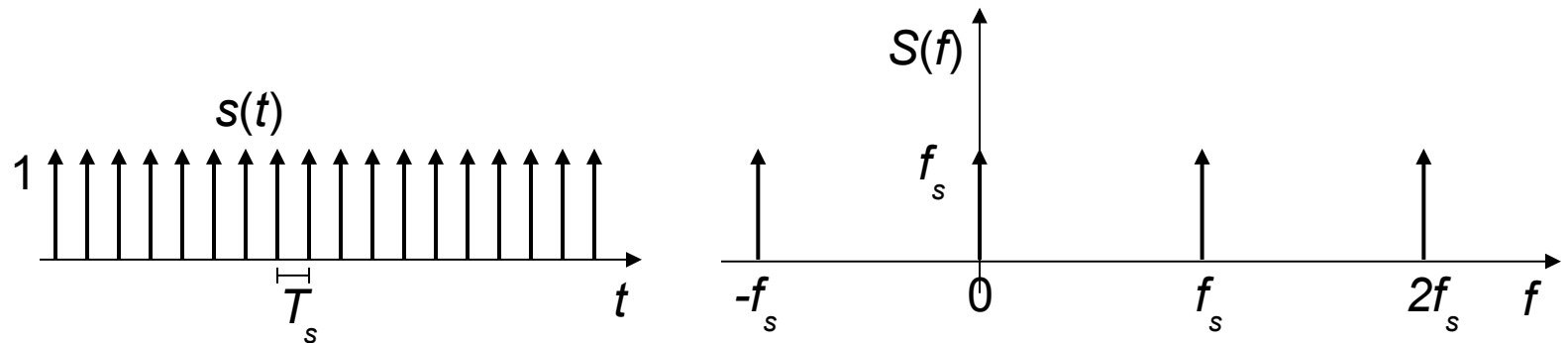


$$x_s(t) = x(t)s(t) = x(t) \sum_{i=-\infty}^{+\infty} \delta(t - iT_s) = \sum_{i=-\infty}^{+\infty} x(iT_s) \delta(t - iT_s)$$

## Segnali campionati



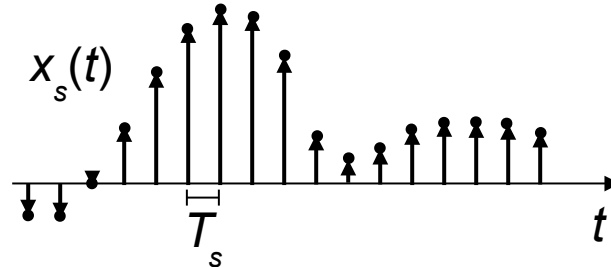
$$x(t) \Leftrightarrow X(f)$$



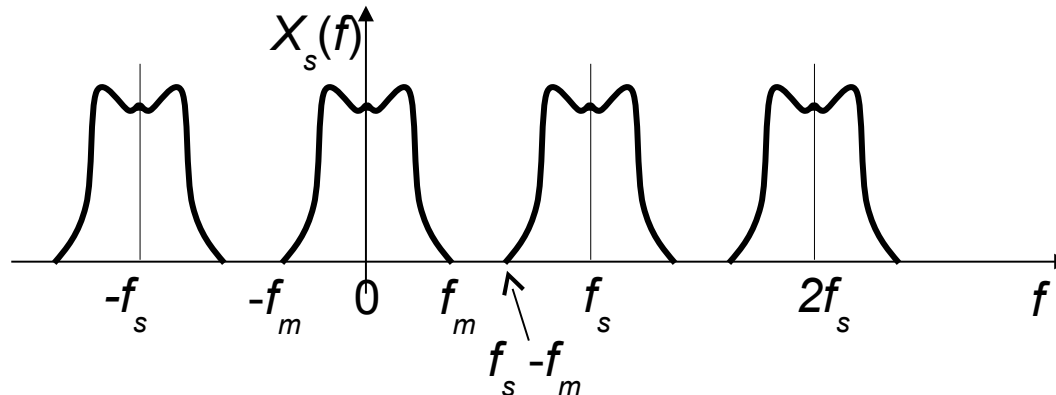
$$s(t) = \sum_{i=-\infty}^{+\infty} \delta(t - iT_s) \Leftrightarrow S(f) = f_s \sum_{k=-\infty}^{+\infty} \delta(f - kf_s)$$

## Campionamento ideale

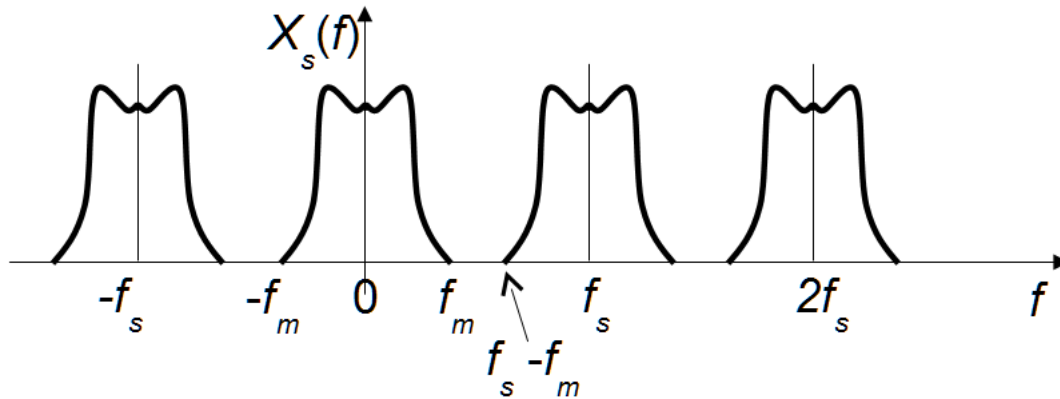
$$x_s(t) = x(t) \cdot s(t) = x(t) \cdot \sum_{i=-\infty}^{+\infty} \delta(t - iT_s)$$



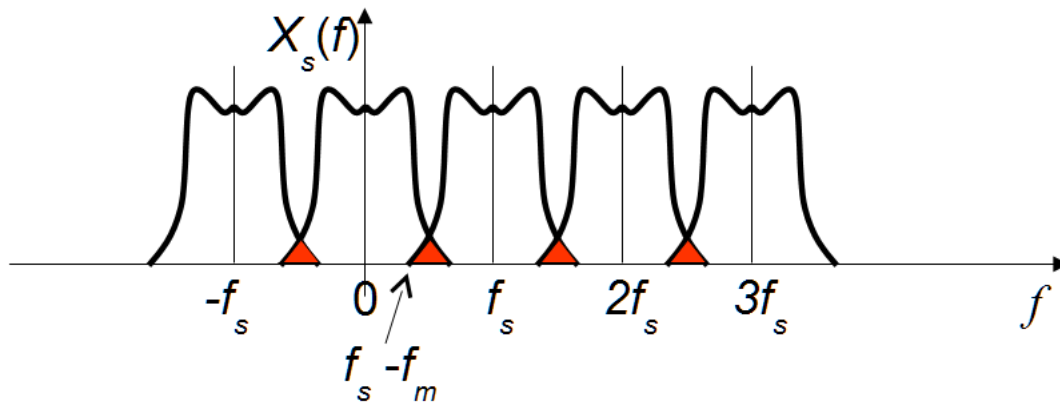
$$X_s(f) = X(f) * S(f) = X(f) * f_s \sum_{k=-\infty}^{+\infty} \delta(f - kf_s) = f_s \sum_{k=-\infty}^{+\infty} X(f - kf_s)$$



## Il teorema del campionamento



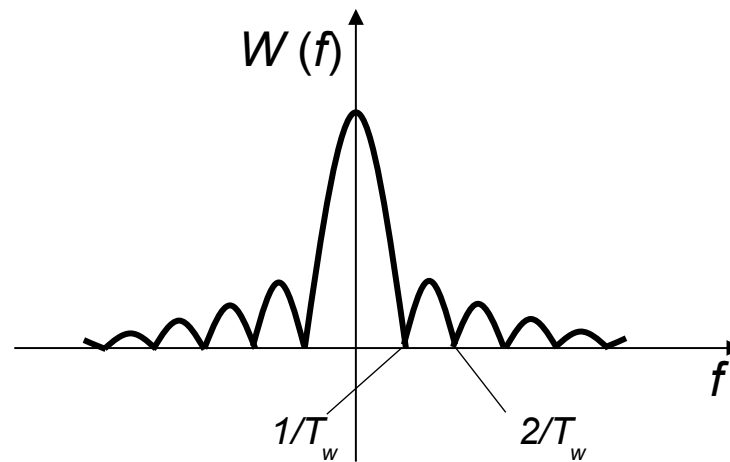
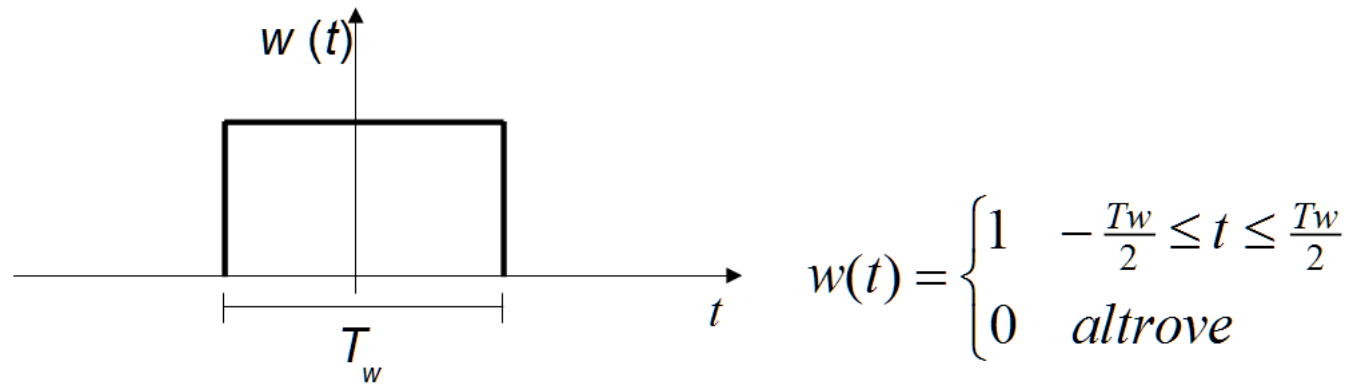
$$f_s = \frac{1}{T_s} \geq 2f_m$$



$$f_s = \frac{1}{T_s} < 2f_m$$

**Aliasing**

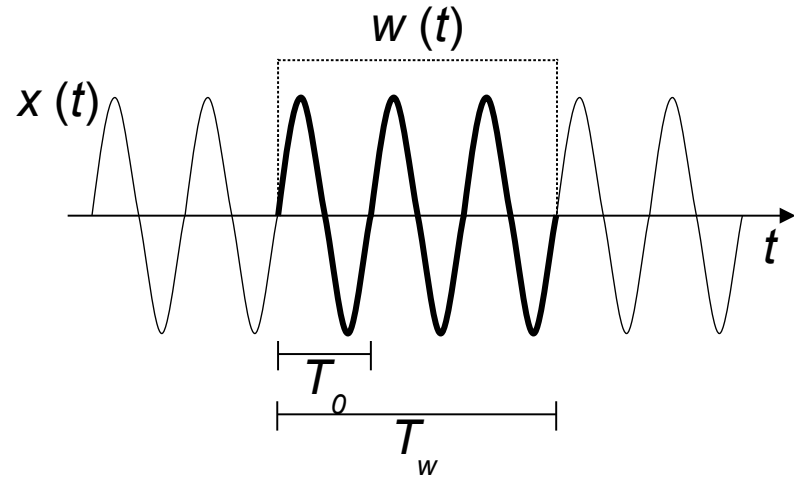
## Troncamento del segnale: finestra rettangolare



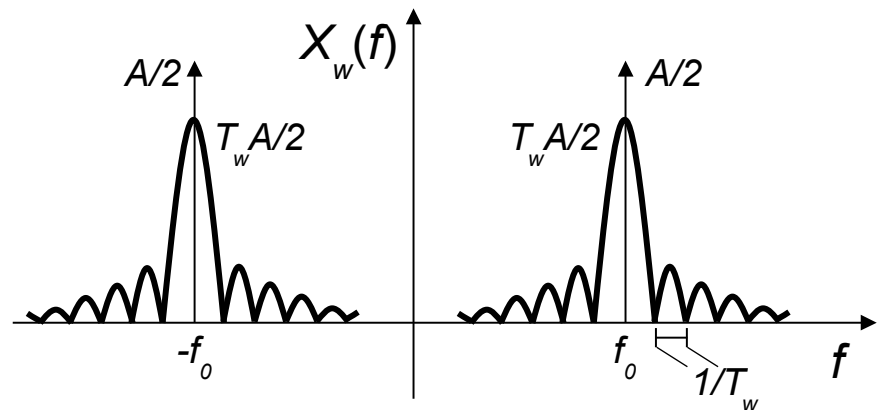


## Troncamento del segnale – *spectral leakage*

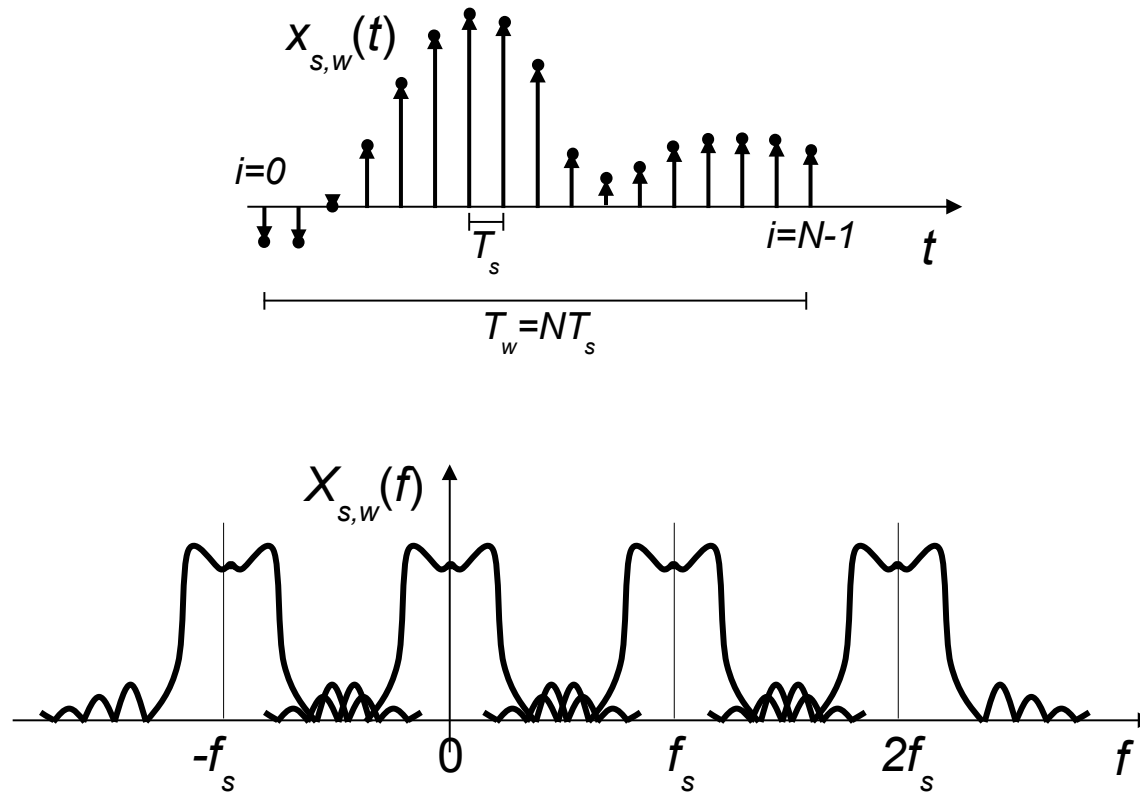
$$x_w(t) = x(t) \cdot w(t)$$



$$X_w(f) = X(f) * W(f)$$

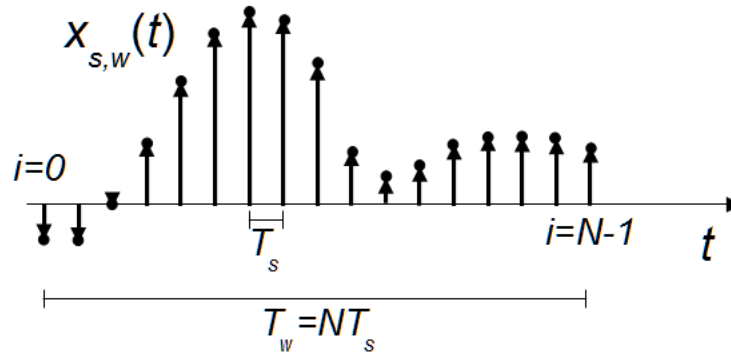


## Segnale campionato e troncato

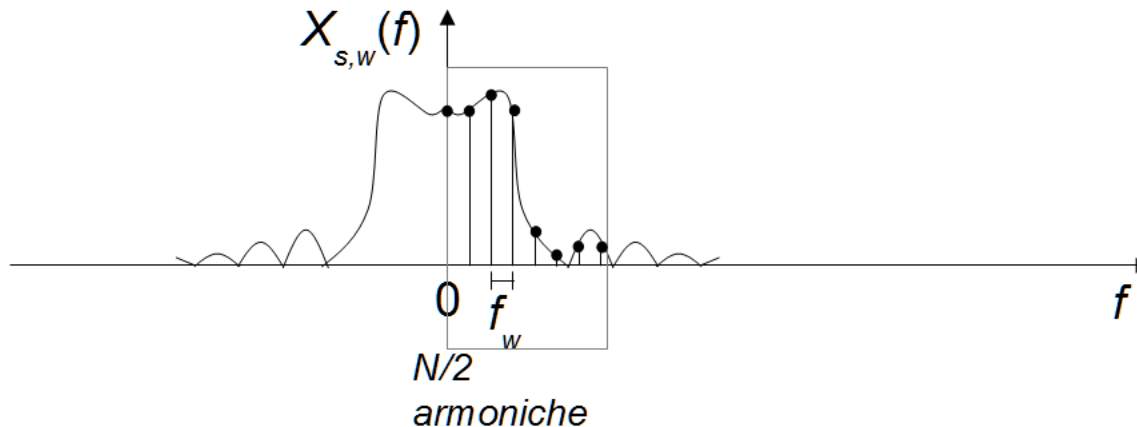


*leakage + aliasing*

## Trasformata discreta di Fourier (DFT)



$$X_k = \sum_{i=0}^{N-1} x(iT_s) e^{-j2\pi k f_w iT_s} = \sum_{i=0}^{N-1} x(iT_s) e^{-j\frac{2\pi}{N} k i} \quad \left( \text{con } f_w = \frac{1}{T_w} = \frac{f_s}{N} \right)$$



## Definizione della DFT

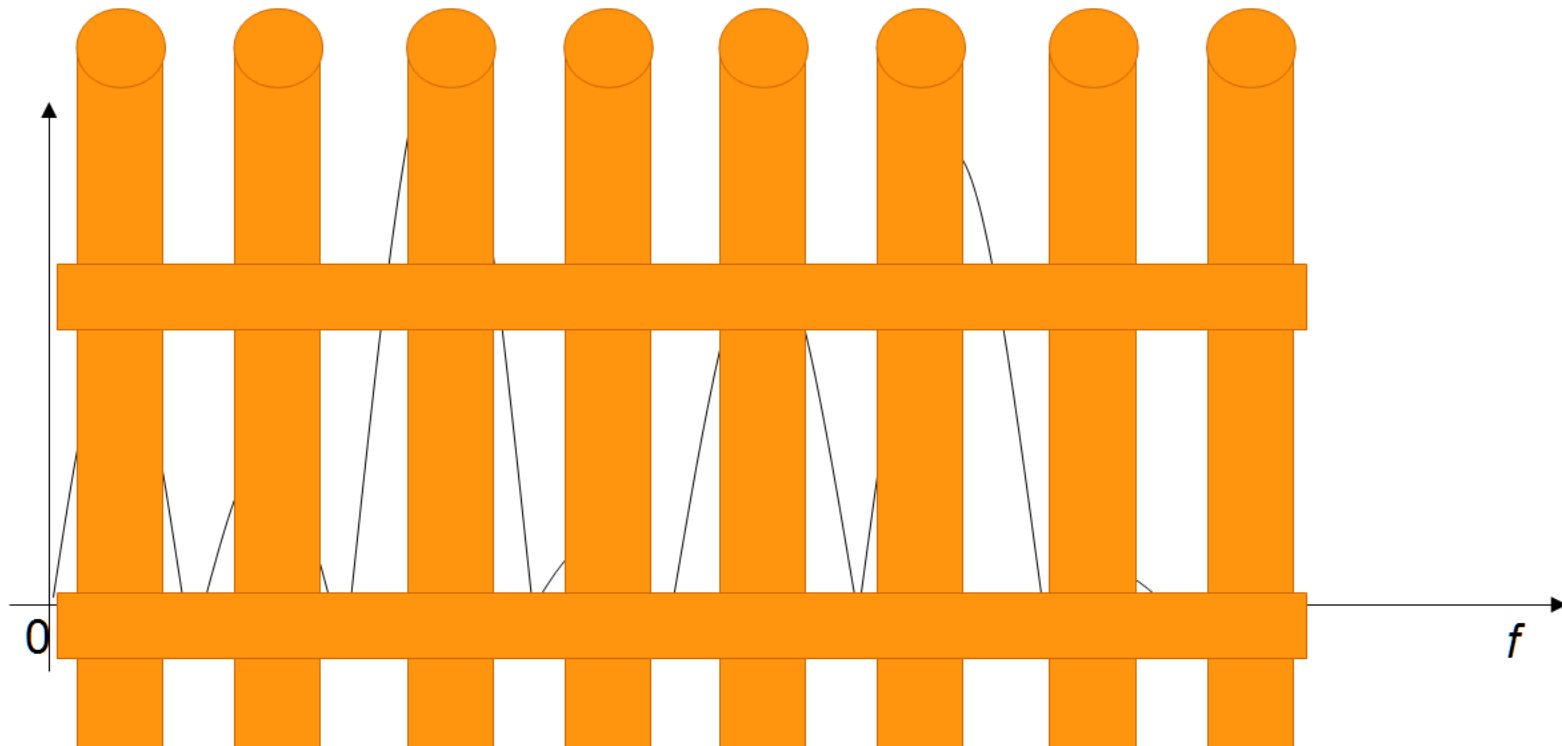
$$X_k = \sum_{i=0}^{N-1} x_i e^{j \frac{-2\pi ki}{N}} \quad (k = 0, 1, 2, \dots, N-1)$$

## Definizione della DFT inversa

$$x_i = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{j \frac{2\pi ki}{N}} \quad (i = 0, 1, 2, \dots, N-1)$$

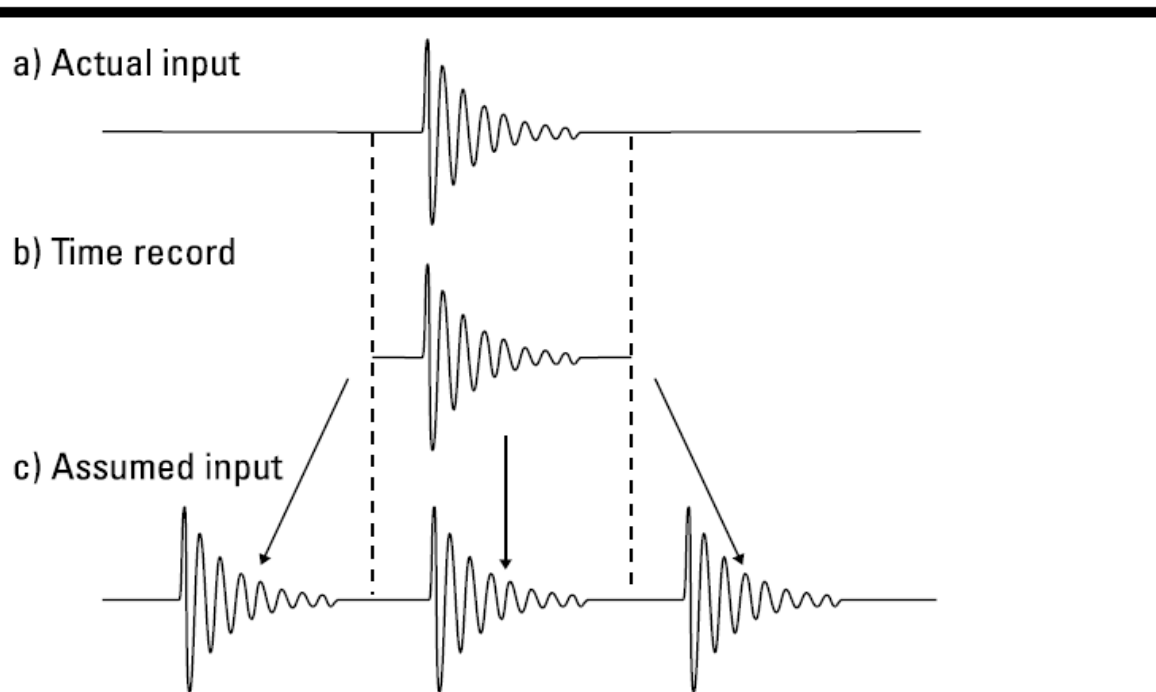
# Trasformata discreta di Fourier (DFT)

## *Picket-fence effect*

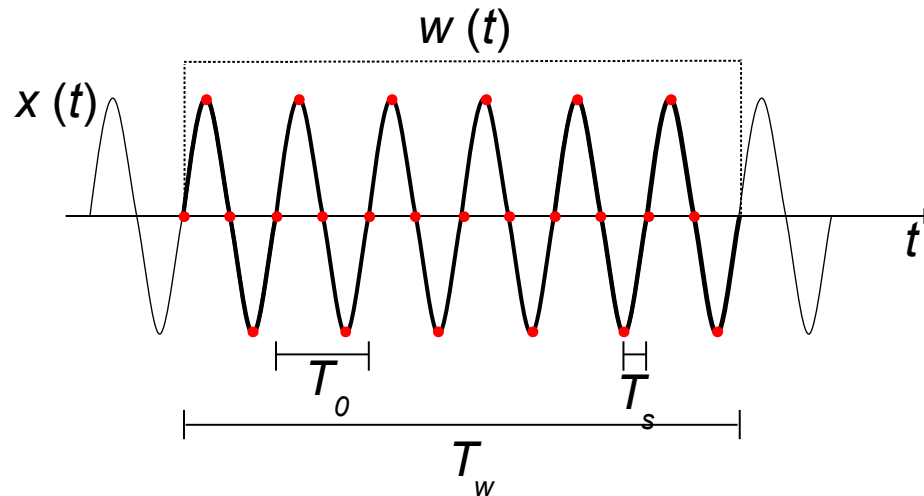


## Principale ipotesi della DFT

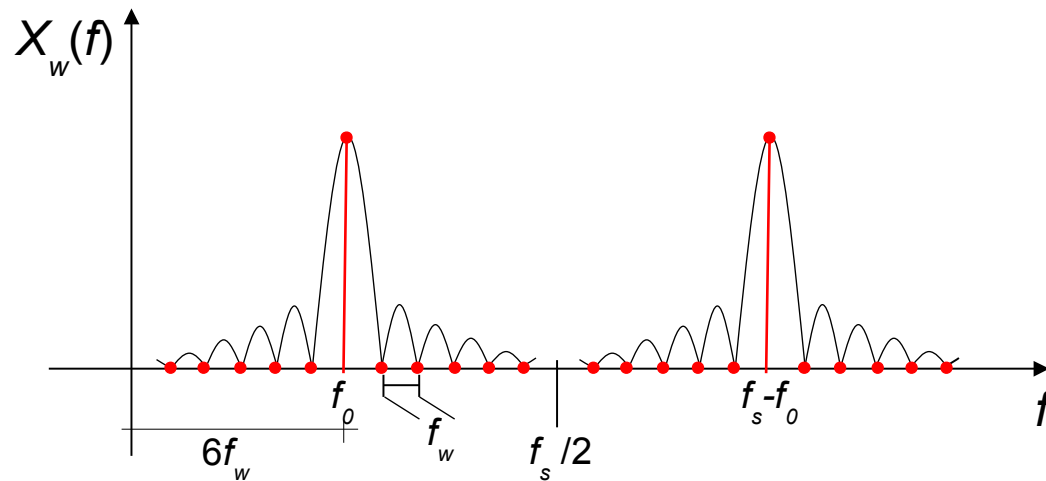
Il segnale di cui la DFT fornisce le righe spettrali deriva dalla replica del *time record* lungo tutto l'asse temporale.



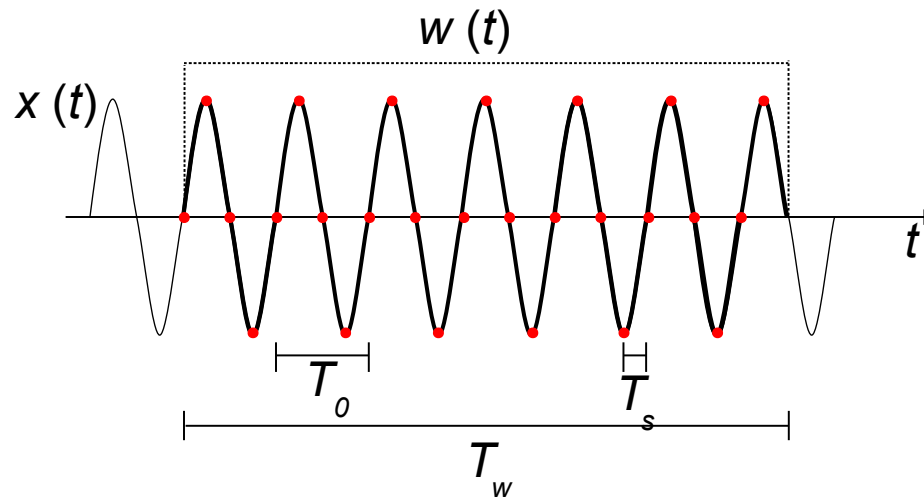
## DFT di segnali periodici (caso 1)



$$T_w = 6T_0 \quad f_s = 24f_w = 4f_0$$

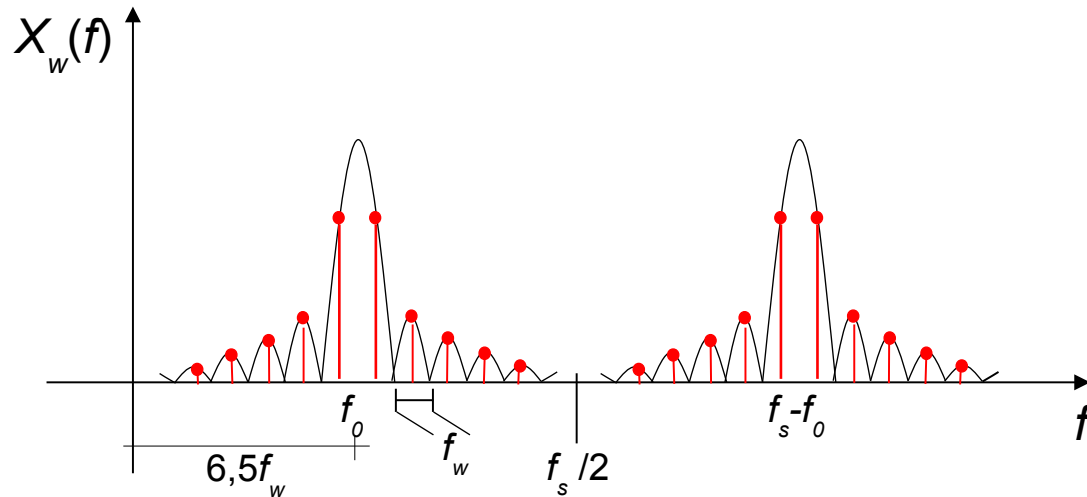


## DFT di segnali periodici (caso 2)



$$T_w = 6,5T_0$$

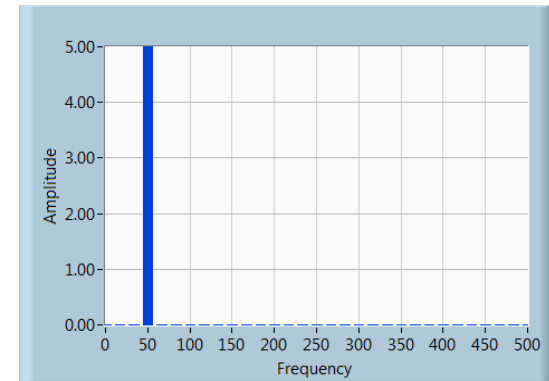
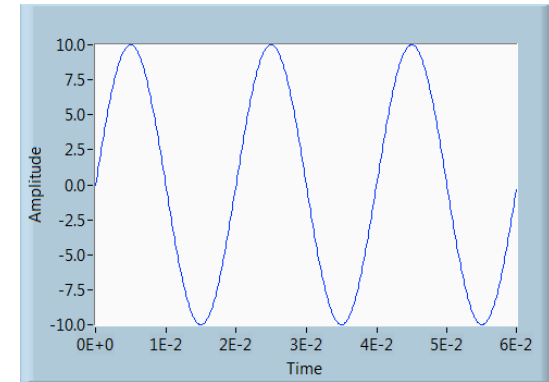
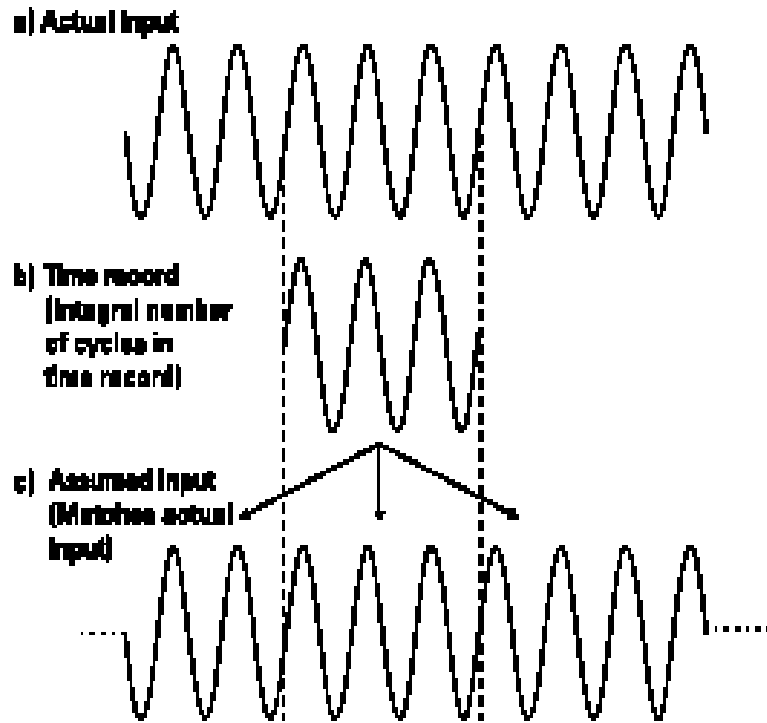
$$f_s = 26f_w = 4f_0$$





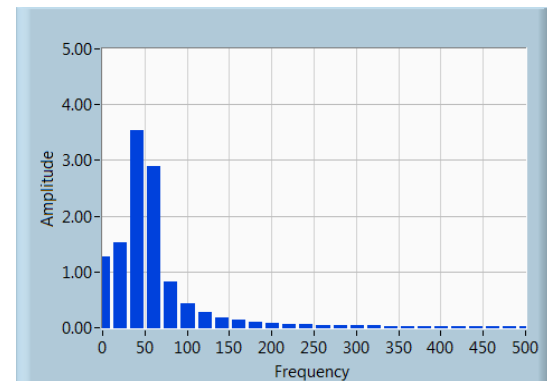
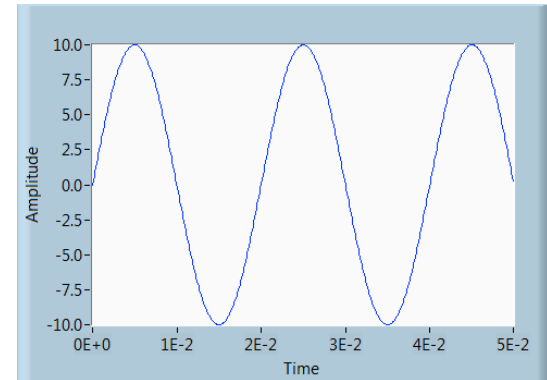
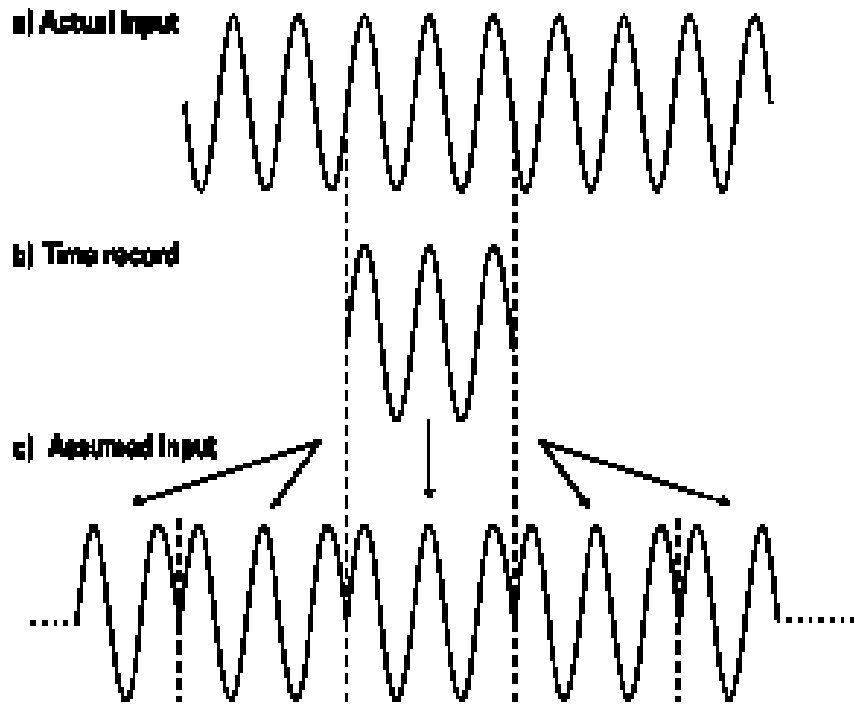
# Spectral Leakage

Segnale periodico nel *time record*



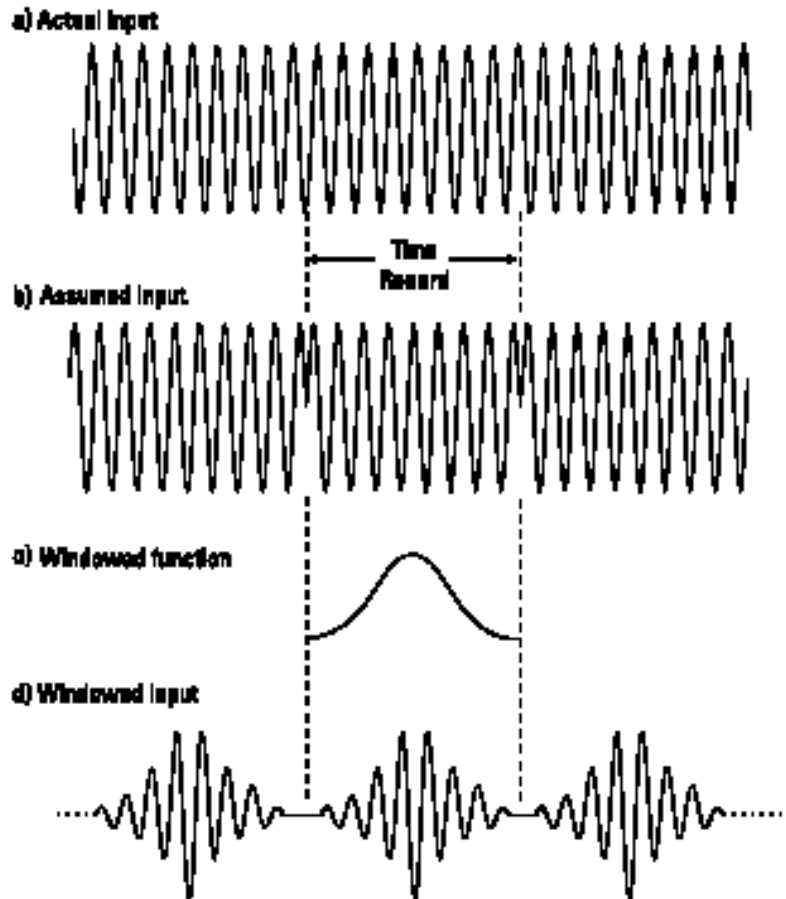
# Spectral Leakage

Segnale non periodico nel *time record*

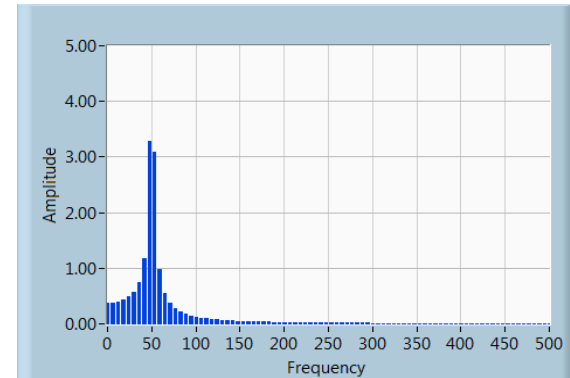


# Finestratura (*Windowing*)

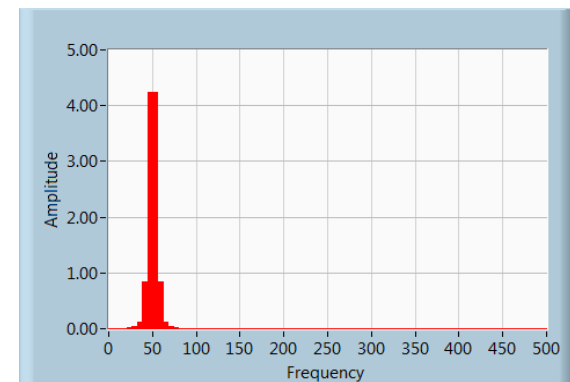
## *Windowing* nel dominio del tempo



## Senza finestra (rettangolare)



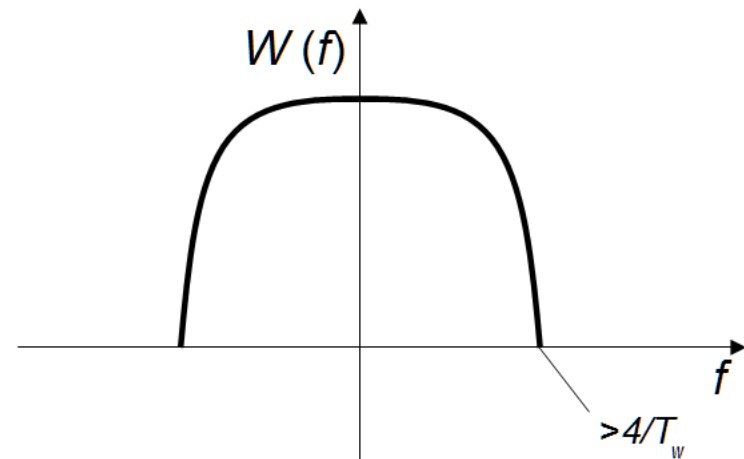
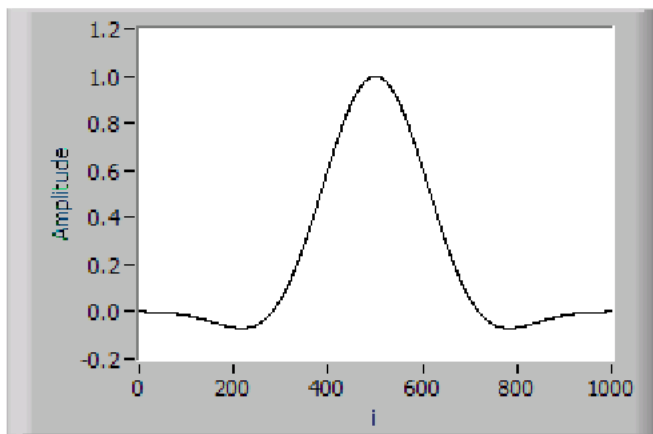
## Con finestra





## Principali finestre: flat top

$$w(i) = 0.2810639 - 0.5208972 \cos\left(\frac{2\pi \cdot i}{N}\right) + 0.1980399 \cos\left(\frac{4\pi \cdot i}{N}\right) \quad i = 0, \dots, N-1$$



## Fast Fourier Transform (FFT)

- ✓ E' un algoritmo per la valutazione della DFT
- ✓  $N$  deve essere una **potenza intera di due**.
- ✓ E' caratterizzata da carico computazionale estremamente ridotto ( **$N \cdot \log N$** :  $N$  numero di campioni) se confrontato con quello derivante dall'applicazione diretta della relazione fondamentale ( **$N^2$** ).

*Esempio: per eseguire una DFT a 512 punti questo algoritmo necessita di 4608 operazioni invece che 262144.*