Hybrid Systems — Homework 3

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Esercise 1. Consider the switched system discussed in class $\{A_1, A_2\}$ with

$$A_1 = \begin{bmatrix} -1 & 10 \\ -100 & -1 \end{bmatrix}; \qquad A_2 = \begin{bmatrix} -1 & 100 \\ -10 & -1 \end{bmatrix}$$

(a) Determine two Lyapunov functions for each linear system A_i $(i =, 1, 2)$ separately. Verify by simulation that these functions are decreasing along an evolution of the system.

(b) Determine if the switched system is quadratically stabilizable.

(c) Determine for each dynamics a minimal dwell time such that the switched system is stable for each switching law $\ell(t)$ that satisfies these minimal dwell times. Show an evolution of the controlled system.

(d) Determine a state feedback stabilizing switching law $\ell(x(t))$ and show an evolution of the controlled system.

(e) Discuss which of the two controlled systems determined in (c) and (d) is faster to reach the origin.

When computing the simulation required in (a), (c) and (d), always use the same initial state $x(0) = (1, 1)$.

Esercise 2. Consider a switched system $\{A_i\}_{i=1,\dots,s}$. Assume that all matrices A_i are stable and that the matrices commute pairwise, i.e, $A_iA_j = A_jA_i$ for all i, j. You should prove that such a system is stable under arbitrary switching.

[Hint: there are several ways to prove this result. If you cannot find an easier proof, you may try this. Define $P_1, \dots, P_s$ as the unique symmetric positive definite matrices that satisfy the Lyapunov equations:

$$\begin{aligned} A_1^T P_1 + P_1 A_1 &= -I, \\ A_i^T P_i + P_i A_i &= -P_{i-1}, \qquad \forall i = 2, \dots, s. \end{aligned}$$

Then show that the quadratic function $V(x) = x^T P_s x$ is a common Lyapunov function for the switched system.

It is sufficient if you prove this result for $s = 3$. If you cannot find a formal proof, verify this result on a switched system of your choice.]

Esercise 3. Consider the switched system $\{A_1, A_2\}$ with

$$A_1 = \begin{bmatrix} 0 & 2 \\ 1 & -2 \end{bmatrix} \quad \text{and} \quad A_2 = \begin{bmatrix} -2 & -3 \\ -3 & 2 \end{bmatrix}.$$

(a) Show that the system is quadratically stabilizable finding an equivalent stable dynamics A_{eq} and its corresponding state feedback switching law $\ell(x(t))$ that stabilizes the system (a chattering free law is required).

(b) Draw the regions of the state space where each dynamics is active and show the system evolution for three different initial conditions.